Name: …………………………………………..

Cargilfield Maths Revision – Book 1

Numbers

Squares: 1   4   9   16   …
(1 x 1 = 1²)   (2 x 2 = 2²)   (3 x 3 = 3²)   (4 x 4 = 4²)

Cubes: 1   8   27   64 …
(1 x 1 x 1 = 1³)   (2 x 2 x 2 = 2³)   (3 x 3 x 3 = 3³)   (4 x 4 x 4 = 4³)

Multiples: Multiply to get multiples.
Example: Multiples of 12: 12, 24, 36, 48, 60, 72 …

Factors: Numbers that go exactly into a larger number.
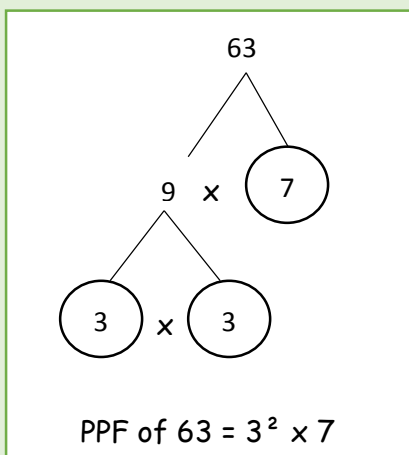Example:

Factors of 12:
1 x 12
2 x 6
3 x 4

Factors of 12: 1, 2, 3, 4, 6, 12

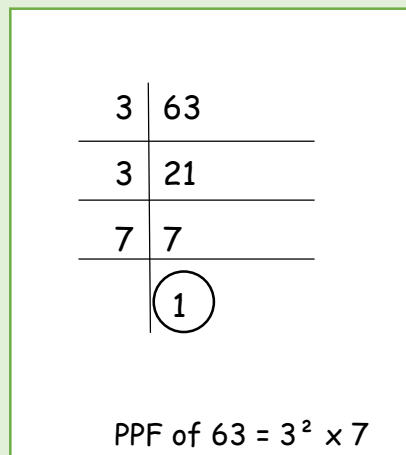**Primes**: Are only divisible by self and 1:   2, 3, 5, 7, 11, 13, 17, 19 …

Product of Prime Factors:

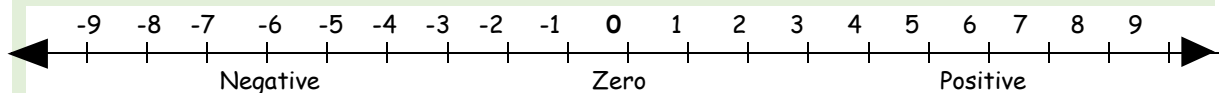Examples: Find the product of prime factors of 63

Method 1:   Method 2

63

9   ×   7

3   ×   3

PPF of 63 = 3² x 7

3	63
3	21
7	7
	1

PPF of 63 = 3² x 7

Negative Numbers

Addition and Subtraction

- Use the number line:

```
-9  -8  -7  -6  -5  -4  -3  -2  -1   0   1   2   3   4   5   6   7   8   9
         Negative                      Zero                   Positive
```

-4 – 3	5 – 11	-4 + 10
= -7	= -6	= 6

- When two negative signs are together they become a positive.
- When a positive and negative sign are together, they become a negative.

8 – (-3)	3 + -(4)	-7 – (-4)
= 8 + 3	= 3 – 4	= -7 + 4
= 11	= -1	= -3

Multiplication and Division

- A negative number multiplied or divided by another negative number gives a positive result.
- A positive number multiplied or divided by a negative number gives a negative result.
- A negative number multiplied or divided by a positive number gives a negative result.

5 x -2	-5 x -10	$(-6)^2$
= - 10	= 50	= -6 x -6
		= 36

-6 ÷ 2	-12 ÷ (-12)	$\dfrac{-16}{7}$
= -3	= 1	$= -2\dfrac{2}{7}$

Roots

Square root:
- The square root of 9 is 3 (since 3 x 3 = 9). This is shown as: $\sqrt{9}$
- The square root of **64** is 8 (since 8 x 8 = 64). $\sqrt{64} = 8$
- $\sqrt{100} = 10,$ $\sqrt{225} = 15,$ $\sqrt{16} = 4$

Cube root:
- The cube root of 27 is 3 (since 3 x 3 x 3 = 27). This is shown as: $\sqrt[3]{27}$
- The cube root of 8 is 2 (since 2 x 2 x 2 = 8). $\sqrt[3]{8}$
- $\sqrt[3]{64} = 4,$ $\sqrt[3]{125} = 5,$ $\sqrt[3]{1000} = 10$

Order of Operations (BIODMAS)

- **BIODMAS** is used to help us decide which operator to use first if there is more than one operator in a calculation.

B – Brackets
I – Index Numbers (e.g. 4^3, 5^2)

O – Of
D – Divide   } Same precedence (work out *left to right*)
M – Multiply

A – Add
S - Subtract   } Same precedence (work out *left to right*)

Examples:

$$4 + \underline{3 \times 2}$$
$$= \quad 4 + \quad 6$$
$$= \quad \underline{10}$$

$$3^2 \div \underline{(12 - 9)}$$
$$= \underline{3^2} \div \quad 3$$
$$= \quad 9 \div 3$$
$$= \quad \underline{3}$$

$$\underline{6 - 3} + 4$$
$$= \quad 3 \quad + 4$$
$$= \quad \underline{7}$$

$$\underline{4 \times 9} + \underline{10 \div 5}$$
$$= \quad 36 \quad + \quad 2$$
$$= \quad \underline{38}$$

$$2 \times 3^2$$
$$= 2 \times 3 \times 3$$
$$= \underline{18}$$

3

Decimals

Addition

132.14 + 39.5

```
  132.14
+  39.50
  171.64
      1
```

Subtraction

282 - 59.5

```
  28²2.¹0
    6 10
-   5̶9̶.5
  222.5
```

Multiplication

21.5 × 0.04

$21.\hat{5} \times 0.0\hat{4}$ (3 hats)

```
    215
×     4
    860
      2
```

$0.\hat{8}\hat{6}\hat{0}$ (3 hats)

= 0.86

Division

8.3 ÷ 0.03
×100   ×100

830 ÷ 3

```
     276.66
3)8²3³0.²0²0
```

= 276.6̇

Multiplication Using Hops

0.35 × 1000

0.350   (3 hops)

= 350

Division Using Hops

46.3 ÷ 100

46.3   (2 hops)

= 0.463

Fractions, Decimals and Percentages

Example: Find 30% of £7.50

$$10\% = 7.50 \div 10$$
$$= 0.75$$

$$30\% = 0.75 \times 3$$
$$= £2.25$$

Example: Find $\frac{3}{5}$ of 45

$$\frac{1}{5} \text{ of } 45 = 45 \div 5$$
$$= 9$$

$$\frac{3}{5} \text{ of } 45 = 9 \times 3$$
$$= 27$$

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{3}{10}$	0.3	30%
$\frac{1}{5}$	0.2	20%
$\frac{1}{3}$	$0.\dot{3}$	$33.\dot{3}\%$
$\frac{2}{3}$	$0.\dot{6}$	$66.\dot{6}\%$

🕐 Change Decimal to Percentage: Multiply by 100.

e.g. $0.83 \times 100 = 83\%$

🕐 Change Percentage to Decimal: Divide by 100.

e.g. $59\% \div 100 = 0.59$

🕐 Change Fraction to Decimal: Divide numerator by denominator.

e.g. $\frac{2}{5} = \frac{4}{10} = 0.4$ e.g. $\frac{13}{20} \overset{\times 5}{\underset{\times 5}{=}} \frac{65}{100} = 0.65$

🕐 Change Decimal to Fraction: Put it out of 10, 100, 1000 and then into lowest terms:

e.g. $0.15 = 0.\overset{\frac{1}{10}}{1}\overset{\frac{1}{100}}{5} = \frac{15}{100} = \frac{3}{20}$

Averages and Data Handling

Consider these scores:

3   6   3   3   5   4

Range = largest – smallest   =   6 – 3 = **3**

Mode = most often occurring score = **3**

Median = line up in order of size and find the middle:

3   3   (3   4)   5   6

= **3.5**

Mean =   total ÷ number of scores
= 24 ÷ 6
= **4**

Probability

- Probability is the chance of something happening.
- It is usually given as a fraction or a decimal.

Mark has the following balls are in a bag:

(B) (B) (B) (R) (R) (Y) (Y) (Y) (Y) (Y)

(b – blue   y – yellow   r – red)

What is the probability of Mark taking out a
i) red ball?

2 out of 10 are red   = $\frac{2}{10}$ = $\frac{1}{5}$

ii) not yellow?

5 balls are not yellow = $\frac{5}{10}$ = $\frac{1}{2}$

Direct Proportion

Jack buys 5 chocolate bars for 76p each. How much does he spend altogether?

$\times 5$ — 1 chocolate bar costs 76p — $\times 5$

5 chocolate bars cost 380p
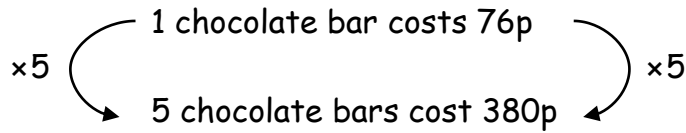
<u>John spends £3.80 altogether</u>

Molly buys pays £5.04 for nine pencils.
What is the cost of 4 pencils?

$\div 9$ — 9 pencils cost 504p — $\div 9$

1 pencil costs 56p

$\times 4$ — 4 pencils cost 224p — $\times 4$
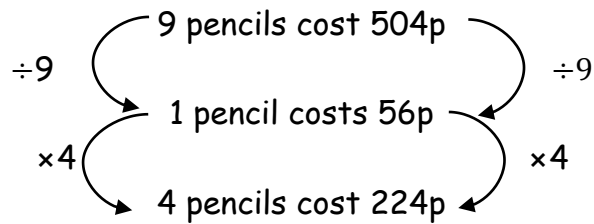
<u>Four pencils cost £2.24</u>

Inverse Proportion

It takes 4 builders 12 days to build a wall. How long would it take 6 builders to build the same wall?

$\div 2$ — 4 builders take 12 days — $\times 2$

2 builders take 24 days

$\times 3$ — 6 builders take 8 days — $\div 3$
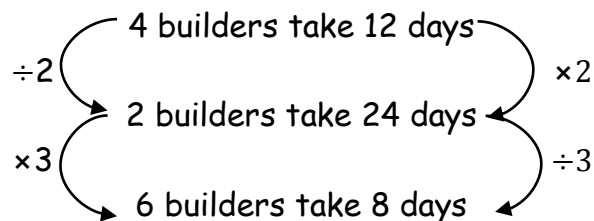
<u>6 builders would take 8 days.</u>   (More hands make light work!)

Ratio

Lowest terms

50 : 65 ÷5 ÷5 **10 : 13**	1.5 : 3.5 x10 x10 = 15 : 35 ÷5 ÷5 = **3 : 7**	$\frac{1}{3} : \frac{3}{4}$ = $\frac{4}{12} : \frac{9}{12}$ = **4 : 9**

Splitting Quantities

Jack and Jill share 42 litres of water in a ratio of 4:3. How much does each person get?

1 division = 42 ÷ 7
       = 6 litres

Jack   Jill

6 litres

7 divisions = 42 litres
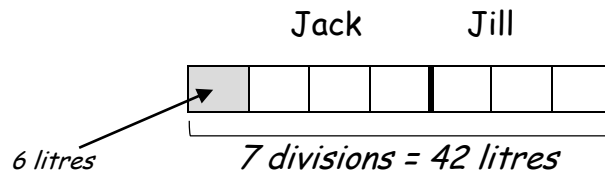
3 divisions = 6 x 3
       = 18 litres

4 division = 6 x 4
       = 24 litres

<u>Jack gets 24 litres, Jill gets 18 litres.</u>

Pale purple paint is mixed using blue, red and white paint in the ratio 2:3:5. If 150ml of red paint is used, how much white paint is used?

     B R W
     2 : 3 : 5
         x50
   ?   150   ?

         B R W
         2 : 3 : 5
    x50 x50 x50
    100   150   250

<u>There is 250ml of white paint used.</u>

Time

There are:
- 🕐 60 seconds in a minute.
- 🕐 60 minutes in an hour.
- 🕐 24 hours in a day.
- 🕐 365 days in a year.
- 🕐 366 days in a leap year.

Length and Distance

Metric Units:
   There are:
- ✈ 10 millimetres in one centimetre
- ✈ 100cm in a metre.
- ✈ 1000m in a kilometre.

Metric and Imperial Conversion:
- ✈ 1 mile is approximately 1609m.
- ✈ 1 inch is approximately 2.54cm.
- ✈ 1 yard is approximately 91cm
- ✈ 1 foot ≈ 30cm

Capacity and Volume

Metric Units:
   There are:
- 🛢 1000ml in a litre
- 🛢 $1cm^3 = 1ml$
- 🛢 $1m^3 = 1,000,000cm^3$ $(1 \times 100 \times 100 \times 100$ *or* $1 \times 100^3)$

Imperial and Metric Conversion:
- 🛢 1 litre ≈ 1.76 pints
- 🛢 1 pint ≈ 0.57 litres
- 🛢 1 gallon ≈ 4.55 litres

Two Dimensional Shapes (Plane Shapes)

Triangles
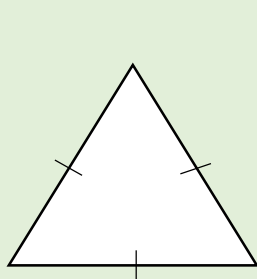
Equilateral Triangle
‣ All sides equal length
‣ All angles the same size
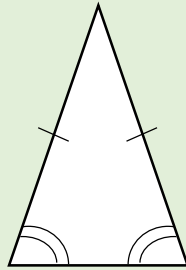
Isosceles Triangle
‣ Two sides equal length
‣ Two angles the same size
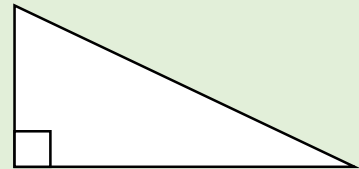
Right-angled Triangle
‣ One angle is 90º

Quadrilaterals
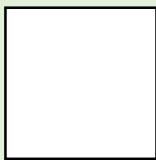
Square

Rectangle
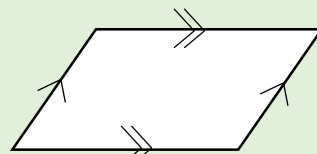
Parallelogram
♦ Two pairs of parallel lines
♦ Opposite angles equal
♦ Opposite sides the same length
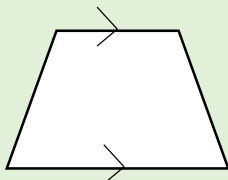
Trapezium
♦ A pair of parallel lines
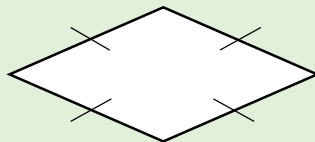
Rhombus
♦ All four sides the same length
♦ Opposite angles equal
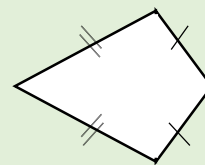
Kite
♦ Two pairs of equal sides

Polygons
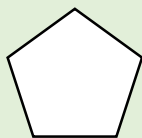
Pentagon
(5 sides)
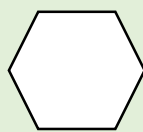
Hexagon
(6 sides)

Heptagon
(7 sides)
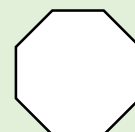
Octagon
(8 sides)

9 sides – nonagon

10 sides – decagon

11 sides – hendecagon

12 sides - dodecagon

Area

♦ Area is the size of a surface

♦ Area of Rectangle = length x breadth

breadth

length
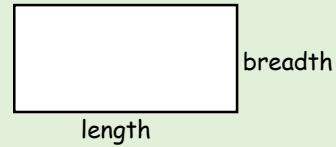
♦ Area of Triangle = ½ base x perpendicular height
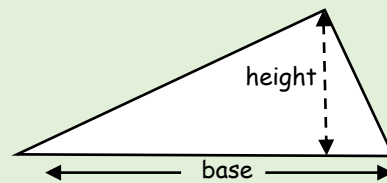
height

base

♦ Area of Parallelogram = base x perpendicular height

height

base
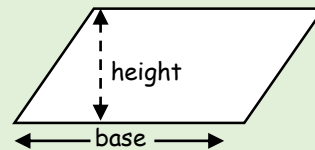
Perimeter

♦ Perimeter is the distance around the edge of a shape.

e.g. Find the perimeter of this rectangle:

8cm

12cm
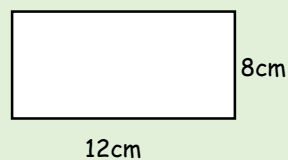
Perimeter = 12 + 12 + 8 + 8
= <u>40cm</u>

3 Dimensional Shapes (Solid Shapes)
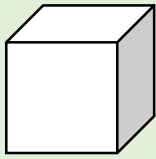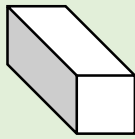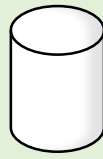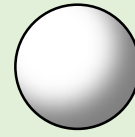
Cube	Cuboid	Cylinder	Sphere
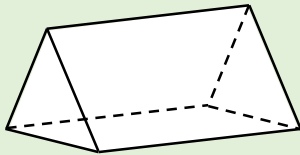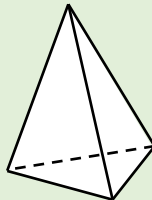
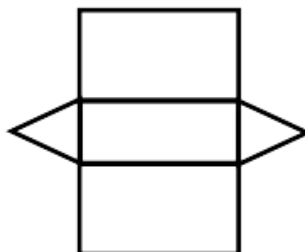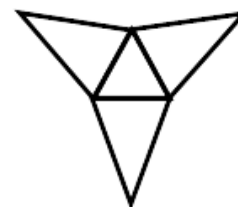|		
Triangular Prism	Tetrahedron (triangle-based pyramid)	Square-based pyramid
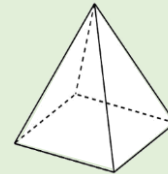
Nets of Solid Shapes
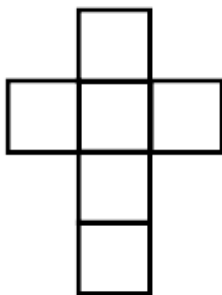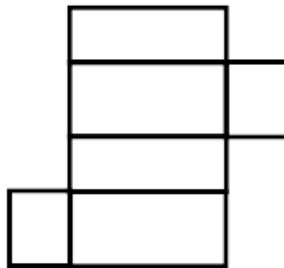
Cube	Cuboid	Cylinder
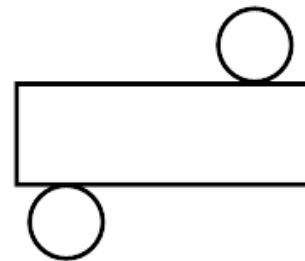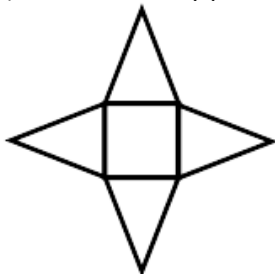
Square-based pyramid	Triangular Prism	Tetrahedron

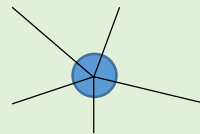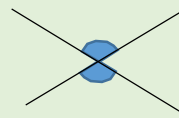## Angles

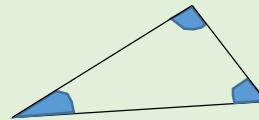- Angles on a straight line have a sum of 180°
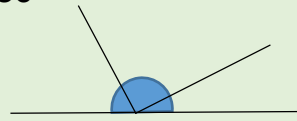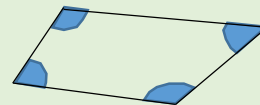
- Angles at a point have a sum of 360°

- Vertically opposite angles are equal

- Interior angles in a triangle have a sum of 180°
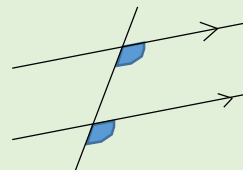
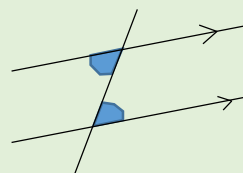- Interior angles in a quadrilateral have a sum of 360°

Parallel Lines

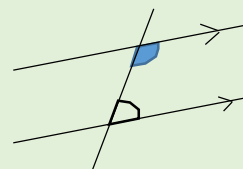Parallel lines run side by side and will never meet

- Corresponding angles are the same

- Alternate angles are the same size

- Co-interior angles have a sum of 180°

Simplification

Examples:

$5x + 3x$ $= \underline{8x}$	$2p - 12p$ $= \underline{-10p}$

$3x \underline{-2y} - 4x \underline{-2y}$ Listing $\cancel{+}x$ $-y$ $\cancel{\searrow}x$ $-y$ $\cancel{+}x$ $-y$ $\cancel{\searrow}x$ $-y$ $\cancel{+}x$ $\cancel{\searrow}x$ $-x$ $= \underline{-x - 4y}$	$3m^2 - 3m + 3$ $= \underline{3m^2 - 3m + 3}$ (no further simplification is possible)

$3 \times y$ $= \underline{3y}$	$a \times a \times a$ $= \underline{a^3}$

$2b \times 4b$ expand $= 2 \times 4 \times b \times b$ $= \underline{8b^2}$	$(-2b)^2$ $= -2b \times -2b$ $= -2 \times -2 \times b \times b$ $= \underline{4b^2}$

Multiplying out Brackets - Claw!

$3(a + 4)$ $= 3a + 12$	$4 - 3(b - 5)$ $= 4 - 3b + 15$ $= \underline{19 - 3b}$

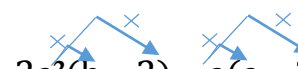|	
$3(a + b) + a(3 + b)$ $= 3a + 3b + 3a + ab$ $= \underline{6a + 3b + ab}$	$2a^2(b - 2) - a(a - 2ab)$ $= 2a^2b - 4a^2 - a^2 + 2a^2b$ $= \underline{4a^2b - 5a^2}$

Equations

Examples:

$b + 3 = 9$
$-3 \quad -3$

$\underline{b = 6}$

$4 - d = 8$
$-4 \quad -4$

$-d = 4$

$\underline{d = -4}$

$3x = 24$

$\dfrac{3x}{3} = \dfrac{24}{3}$

$\underline{x = 8}$

$\dfrac{e}{6} = 12$

$e = 12 \times 6$

$\underline{e = 72}$

12	12	12	12	12	12

e

$3m + 9 = 24$
$\quad -9 \qquad -9$

$\dfrac{3m}{3} = \dfrac{15}{3}$

$\underline{m = 5}$

$4n - 1 = 2$
$\quad +1 \qquad +1$

$\dfrac{4n}{4} = \dfrac{3}{4}$

$n = \dfrac{3}{4}$

Direction and the Compass

North

North West   North East

West   East

South West   South East

South
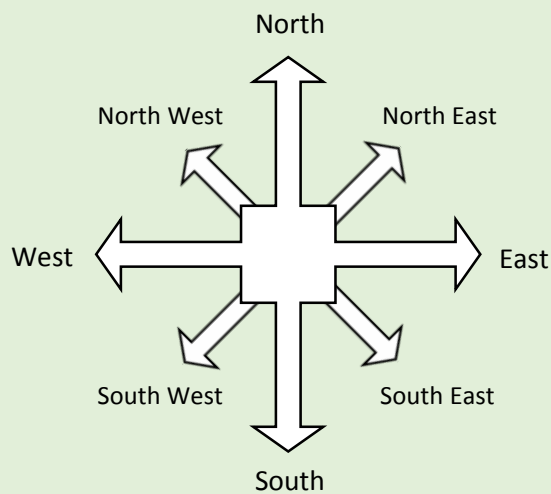
To remember use this mnemonic going clockwise from North:

Never **E**at **S**hredded **W**heat   or   **N**aughty **E**lephants **S**pray **W**ater

- A coordinate is used to specify a particular position on a grid:
- Point A is 3 along and 2 down <u>or</u> A(3,-2)
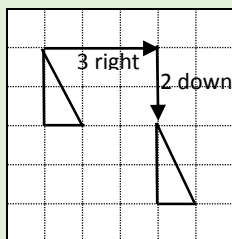- Point B is 2 left and 4 up: B(-2,4)
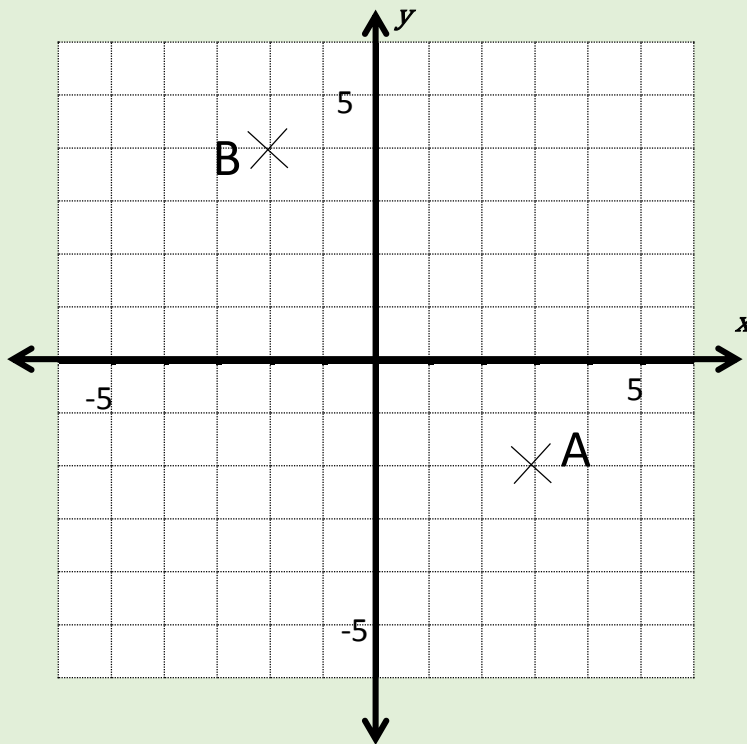
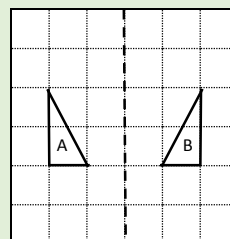## Transformations

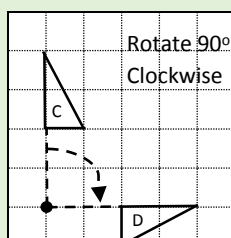- There are 4 types of transformations:

Translation
move the shape:

3 right
2 down

Reflection
Make a mirror image:

A   B

B is a reflection of A

Rotation
Turn the shape:
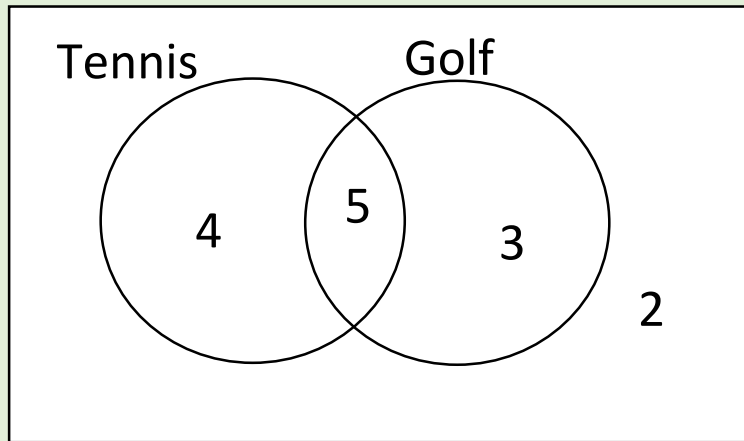
Rotate 90°
Clockwise

C

D

Enlargement
Make the shape larger:
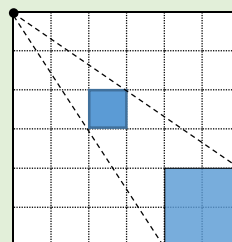
Enlarge shape E using
a scale factor of 2 to
make shape F

- A Venn diagram is used to show information about sets of data.

The Venn diagram below shows the sports played by 14 children:

A total of 8 people play golf

A total of 9 people play tennis

7 people play only one sport

4 people play tennis only | 5 people play both sports | 3 people play golf only | 2 people don't play either sport

Rounding

Decimal Places (d.p.)

- Decimal places are the digits which come after a decimal point.
  Example:
  3.155 has 3 decimal places: 3.1 5 5

- Example of rounding off to decimal places:
  - Round 7.467 to 2 decimal places:

  7.4 6 5

  ≈ 7.47 (the 6 changes since the digit to its right is 5 or greater)

Significant Figures (s.f.)

- Start counting significant figures from the first non-zero.
  3.401 has 4 s.f.   0.044 has 2 s.f.   3183 has 4 s.f.
- Examples of rounding:
  - 351 to 1 s.f. = 400
  - 0.8483 to 3 s.f. = 0.848