

Name:

Cargilfield Maths Revision – Book 1

Numbers

Squares: 1 4 9 16 ...
 (1 × 1 = 1²) (2 × 2 = 2²) (3 × 3 = 3²) (4 × 4 = 4²)

Cubes: 1 8 27 64 ...
 (1 × 1 × 1 = 1³) (2 × 2 × 2 = 2³) (3 × 3 × 3 = 3³) (4 × 4 × 4 = 4³)

Multiples: Multiply to get multiples.
Example: Multiples of 12: 12, 24, 36, 48, 60, 72 ...

Factors: Numbers that go exactly into a larger number.
Example:

Factors of 12:

1 × 12	↑
2 × 6	
3 × 4	

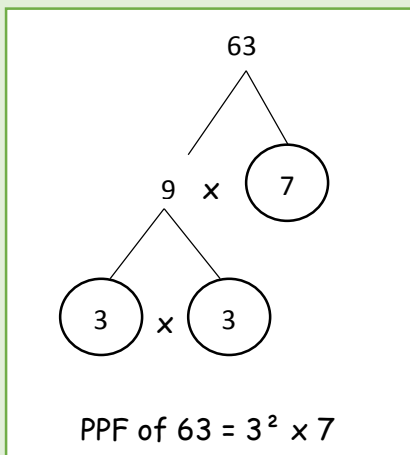
Factors of 12: 1, 2, 3, 4, 6, 12

Primes: Are only divisible by self and 1: 2, 3, 5, 7, 11, 13, 17, 19 ...

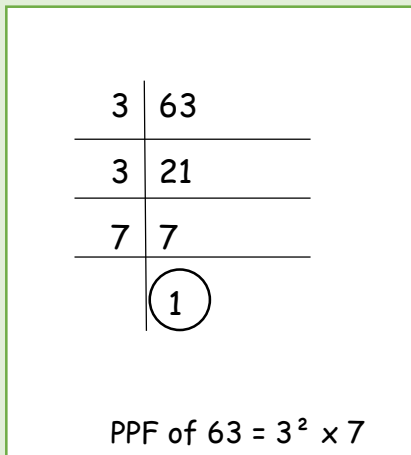
Product of Prime Factors:

Examples: Find the product of prime factors of 63

Method 1:



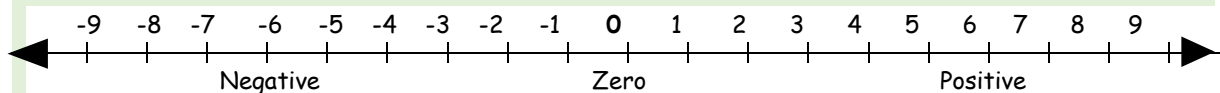
Method 2



Negative Numbers

Addition and Subtraction

- Use the number line:



$$\begin{aligned} -4 - 3 \\ = \underline{-7} \end{aligned}$$

$$\begin{aligned} 5 - 11 \\ = \underline{-6} \end{aligned}$$

$$\begin{aligned} -4 + 10 \\ = \underline{6} \end{aligned}$$

- When two negative signs are together they become a positive.
- When a positive and negative sign are together, they become a negative.

$$\begin{aligned} 8 - (-3) \\ = 8 + 3 \\ = \underline{11} \end{aligned}$$

$$\begin{aligned} 3 + -(4) \\ = 3 - 4 \\ = \underline{-1} \end{aligned}$$

$$\begin{aligned} -7 - (-4) \\ = -7 + 4 \\ = \underline{-3} \end{aligned}$$

Multiplication and Division

- A negative number multiplied or divided by another negative number gives a positive result.
- A positive number multiplied or divided by a negative number gives a negative result.
- A negative number multiplied or divided by a positive number gives a negative result.

$$\begin{aligned} 5 \times -2 \\ = \underline{-10} \end{aligned}$$

$$\begin{aligned} -5 \times -10 \\ = \underline{50} \end{aligned}$$

$$\begin{aligned} (-6)^2 \\ = -6 \times -6 \\ = \underline{36} \end{aligned}$$

$$\begin{aligned} -6 \div 2 \\ = \underline{-3} \end{aligned}$$

$$\begin{aligned} -12 \div (-12) \\ = \underline{1} \end{aligned}$$

$$\begin{aligned} \frac{-16}{7} \\ = -2\frac{2}{7} \end{aligned}$$

Roots

Square root:

- The square root of 9 is 3 (since $3 \times 3 = 9$). This is shown as: $\sqrt{9}$
- The square root of 64 is 8 (since $8 \times 8 = 64$). $\sqrt{64} = 8$
- $\sqrt{100} = 10$, $\sqrt{225} = 15$, $\sqrt{16} = 4$

Cube root:

- The cube root of 27 is 3 (since $3 \times 3 \times 3 = 27$). This is shown as: $\sqrt[3]{27}$
- The cube root of 8 is 2 (since $2 \times 2 \times 2 = 8$). $\sqrt[3]{8}$
- $\sqrt[3]{64} = 4$, $\sqrt[3]{125} = 5$, $\sqrt[3]{1000} = 10$

Order of Operations (BIODMAS)

- **BIODMAS** is used to help us decide which operator to use first if there is more than one operator in a calculation.

B - Brackets

I - Index Numbers (e.g. 4^3 , 5^2)

O - Of

D - Divide

M - Multiply

} Same precedence (work out *left to right*)

A - Add

S - Subtract

} Same precedence (work out *left to right*)

Examples:

$$\begin{aligned} & 4 + \underline{3 \times 2} \\ & = 4 + 6 \\ & = \underline{10} \end{aligned}$$

$$\begin{aligned} & 3^2 \div \underline{(12 - 9)} \\ & = \underline{3^2} \div 3 \\ & = 9 \div 3 \\ & = \underline{3} \end{aligned}$$

$$\begin{aligned} & \underline{6 - 3} + 4 \\ & = 3 + 4 \\ & = \underline{7} \end{aligned}$$

$$\begin{aligned} & \underline{4 \times 9} + \underline{10 \div 5} \\ & = 36 + 2 \\ & = \underline{38} \end{aligned}$$

$$\begin{aligned} & 2 \times 3^2 \\ & = 2 \times 3 \times 3 \\ & = \underline{18} \end{aligned}$$

Decimals

Addition

$$132.14 + 39.5$$

$$\begin{array}{r} 132.14 \\ + 39.50 \\ \hline 171.64 \\ \hline \end{array}$$

Subtraction

$$282 - 59.5$$

Put in a zero!

$$\begin{array}{r} 282.10 \\ - 59.5 \\ \hline 222.5 \\ \hline \end{array}$$

Multiplication

$$21.5 \times 0.04$$

$$21.5 \times 0.04 \quad (3 \text{ hats})$$

$$\begin{array}{r} 215 \\ \times 4 \\ \hline 860 \\ \hline \end{array}$$

$$0.860 \quad (3 \text{ hats})$$

$$= \underline{0.86}$$

Division

$$8.3 \div 0.03$$

x100 x100

$$830 \div 3$$

$$\begin{array}{r} 276.66 \\ 3 \overline{)830.00} \end{array}$$

$$= \underline{276.6}$$

Multiplication Using Hops

$$0.35 \times 1000$$

$$0.350 \quad (3 \text{ hops})$$

$$= \underline{350}$$

Division Using Hops

$$46.3 \div 100$$

$$463 \quad (2 \text{ hops})$$

$$= \underline{0.463}$$

Fractions, Decimals and Percentages

Example: Find 30% of £7.50

$$10\% = 7.50 \div 10 \\ = 0.75$$

$$30\% = 0.75 \times 3 \\ = \underline{\underline{\pounds 2.25}}$$

Example: Find $\frac{3}{5}$ of 45

$$\frac{1}{5} \text{ of } 45 = 45 \div 5 \\ = 9$$

$$\frac{3}{5} \text{ of } 45 = 9 \times 3 \\ = \underline{\underline{27}}$$

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{3}{10}$	0.3	30%
$\frac{1}{5}$	0.2	20%
$\frac{1}{3}$	0. $\dot{3}$	33. $\dot{3}$ %
$\frac{2}{3}$	0. $\dot{6}$	66. $\dot{6}$ %

- ⌚ Change Decimal to Percentage: Multiply by 100.

$$\text{e.g. } 0.83 \times 100 = 83\%$$

- ⌚ Change Percentage to Decimal: Divide by 100.

$$\text{e.g. } 59\% \div 100 = 0.59$$

- ⌚ Change Fraction to Decimal: Divide numerator by denominator.

$$\text{e.g. } \frac{2}{5} = \frac{4}{10} = 0.4$$

$$\text{e.g. } \frac{13}{20} \overset{\times 5}{=} \frac{65}{100} = 0.65$$

- ⌚ Change Decimal to Fraction: Put it out of 10, 100, 1000 and then into lowest terms:

$$\text{e.g. } 0.15 = 0.\overset{\frac{1}{10}}{\underset{\frac{1}{100}}{15}} = \frac{15}{100} = \frac{3}{20}$$

Averages and Data Handling

Consider these scores:

3 6 3 3 5 4

Range = largest - smallest = $6 - 3 = \underline{3}$

Mode = most often occurring score = 3

Median = line up in order of size and find the middle:

3 3 3 4 5 6

= 3.5

Mean = total \div number of scores
= $24 \div 6$
= 4

Probability

- Probability is the chance of something happening.
- It is usually given as a fraction or a decimal.

Mark has the following balls are in a bag:



(*b - blue y - yellow r - red*)

What is the probability of Mark taking out a

i) red ball?

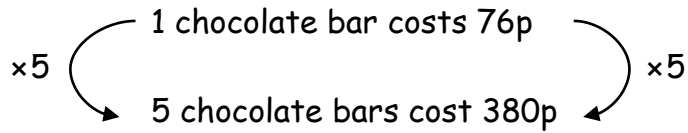
$$2 \text{ out of } 10 \text{ are red} = \frac{2}{10} = \frac{1}{5}$$

ii) not yellow?

$$5 \text{ balls are not yellow} = \frac{5}{10} = \frac{1}{2}$$

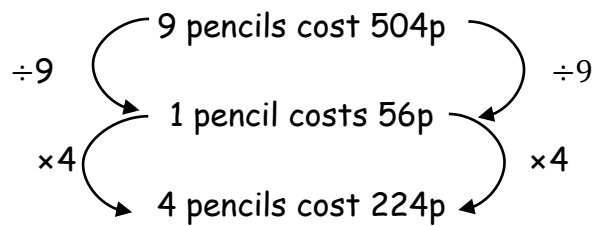
Direct Proportion

Jack buys 5 chocolate bars for 76p each. How much does he spend altogether?



John spends £3.80 altogether

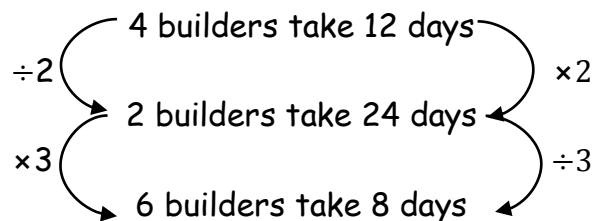
Molly buys pays £5.04 for nine pencils.
What is the cost of 4 pencils?



Four pencils cost £2.24

Inverse Proportion

It takes 4 builders 12 days to build a wall. How long would it take 6 builders to build the same wall?



6 builders would take 8 days. (More hands make light work!)

Ratio

Lowest terms

$$50 : 65$$
$$\div 5 \quad \div 5$$

$$= \underline{10 : 13}$$

$$1.5 : 3.5$$
$$\times 10 \quad \times 10$$

$$= 15 : 35$$
$$\div 5 \quad \div 5$$

$$= \underline{3 : 7}$$

$$\frac{1}{3} : \frac{3}{4}$$

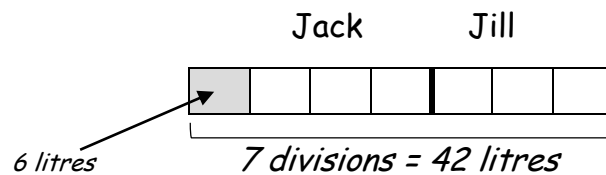
$$= \frac{4}{12} : \frac{9}{12}$$

$$= \underline{4 : 9}$$

Splitting Quantities

Jack and Jill share 42 litres of water in a ratio of 4:3. How much does each person get?

$$1 \text{ division} = 42 \div 7$$
$$= \underline{6 \text{ litres}}$$



$$3 \text{ divisions} = 6 \times 3$$
$$= \underline{18 \text{ litres}}$$

$$4 \text{ divisions} = 6 \times 4$$
$$= \underline{24 \text{ litres}}$$

Jack gets 24 litres, Jill gets 18 litres.

Pale purple paint is mixed using blue, red and white paint in the ratio 2:3:5. If 150ml of red paint is used, how much white paint is used?

$$\begin{array}{ccc} \text{B} & \text{R} & \text{W} \\ 2 & : & 3 & : & 5 \\ \swarrow & & \downarrow & & \searrow \\ ? & & 150 & & ? \end{array}$$

$\times 50$

$$\begin{array}{ccc} \text{B} & \text{R} & \text{W} \\ 2 & : & 3 & : & 5 \\ \swarrow & & \downarrow & & \searrow \\ 100 & & 150 & & 250 \end{array}$$

$\times 50 \quad \times 50 \quad \times 50$

There is 250ml of white paint used.

Units

Time

There are:

- 🕒 60 seconds in a minute.
- 🕒 60 minutes in an hour.
- 🕒 24 hours in a day.
- 🕒 365 days in a year.
- 🕒 366 days in a leap year.

Length and Distance

Metric Units:

There are:

- ✦ 10 millimetres in one centimetre
- ✦ 100cm in a metre.
- ✦ 1000m in a kilometre.

Metric and Imperial Conversion:

- ✦ 1 mile is approximately 1609m.
- ✦ 1 inch is approximately 2.54cm.
- ✦ 1 yard is approximately 91cm
- ✦ 1 foot \approx 30cm

Capacity and Volume

Metric Units:

There are:

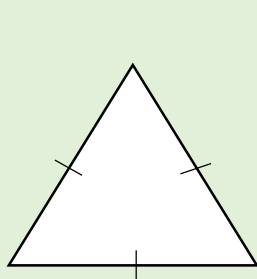
- 🗄 1000ml in a litre
- 🗄 $1\text{cm}^3 = 1\text{ml}$
- 🗄 $1\text{m}^3 = 1,000,000\text{cm}^3$ ($1 \times 100 \times 100 \times 100$ or 1×100^3)

Imperial and Metric Conversion:

- 🗄 1 litre \approx 1.76 pints
- 🗄 1 pint \approx 0.57 litres
- 🗄 1 gallon \approx 4.55 litres

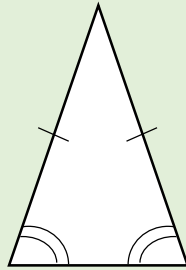
Two Dimensional Shapes (Plane Shapes)

Triangles



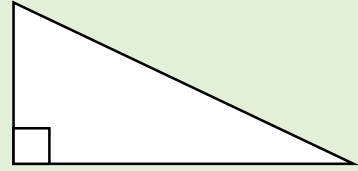
Equilateral Triangle

- ▶ All sides equal length
- ▶ All angles the same size



Isosceles Triangle

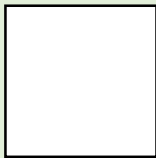
- ▶ Two sides equal length
- ▶ Two angles the same size



Right-angled Triangle

- ▶ One angle is 90°

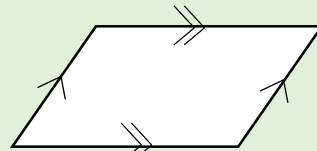
Quadrilaterals



Square

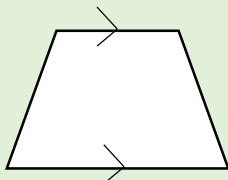


Rectangle



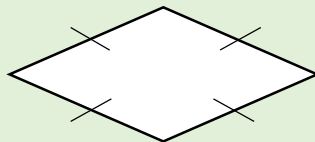
Parallelogram

- ◆ Two pairs of parallel lines
- ◆ Opposite angles equal
- ◆ Opposite sides the same length



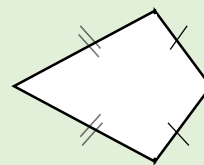
Trapezium

- ◆ A pair of parallel lines



Rhombus

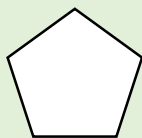
- ◆ All four sides the same length
- ◆ Opposite angles equal



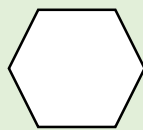
Kite

- ◆ Two pairs of equal sides

Polygons



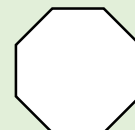
Pentagon
(5 sides)



Hexagon
(6 sides)



Heptagon
(7 sides)



Octagon
(8 sides)

9 sides - nonagon

10 sides - decagon

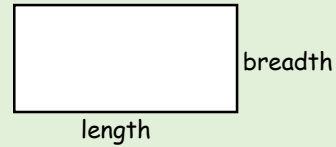
11 sides - hendecagon

12 sides - dodecagon

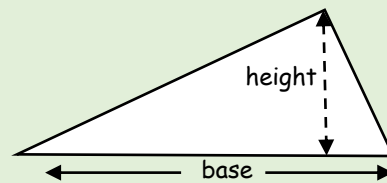
Area

◆ Area is the size of a surface

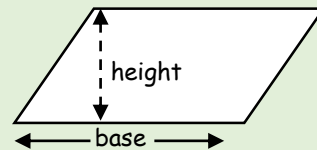
◆ Area of Rectangle = length x breadth



◆ Area of Triangle = $\frac{1}{2}$ base x perpendicular height



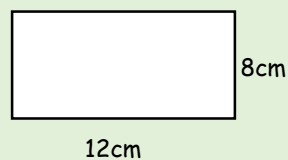
◆ Area of Parallelogram = base x perpendicular height



Perimeter

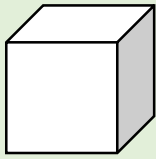
◆ Perimeter is the distance around the edge of a shape.

e.g. Find the perimeter of this rectangle:

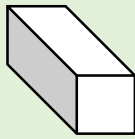


$$\begin{aligned}\text{Perimeter} &= 12 + 12 + 8 + 8 \\ &= \underline{40\text{cm}}\end{aligned}$$

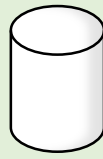
3 Dimensional Shapes (Solid Shapes)



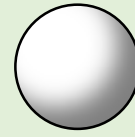
Cube



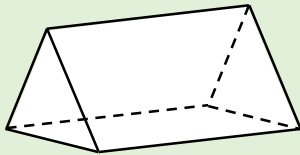
Cuboid



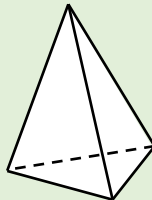
Cylinder



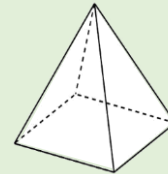
Sphere



Triangular Prism



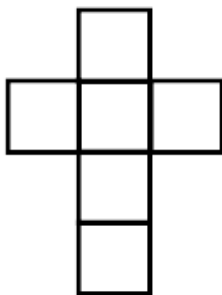
Tetrahedron
(triangle-based pyramid)



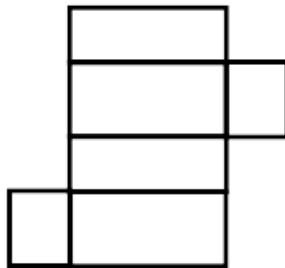
Square-based pyramid

Nets of Solid Shapes

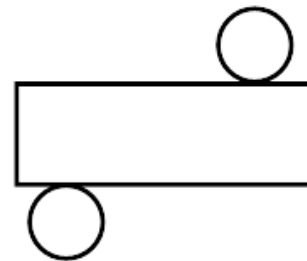
Cube



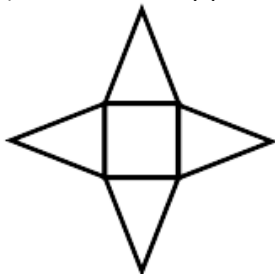
Cuboid



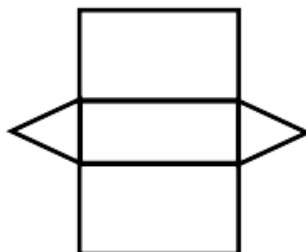
Cylinder



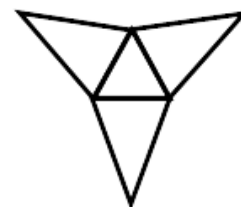
Square-based pyramid



Triangular Prism

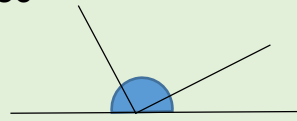


Tetrahedron

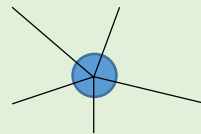


Angles

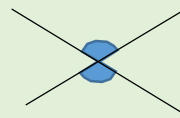
- Angles on a straight line have a sum of 180°



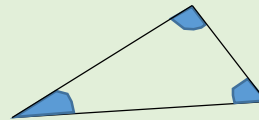
- Angles at a point have a sum of 360°



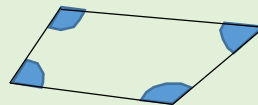
- Vertically opposite angles are equal



- Interior angles in a triangle have a sum of 180°



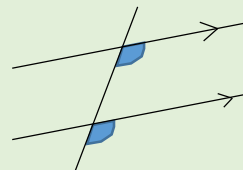
- Interior angles in a quadrilateral have a sum of 360°



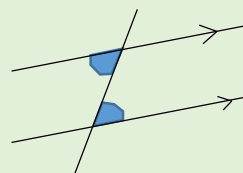
Parallel Lines

Parallel lines run side by side and will never meet

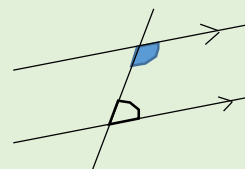
- Corresponding angles are the same



- Alternate angles are the same size



- Co-interior angles have a sum of 180°



Simplification

Examples:

$$5x + 3x$$

$$= \underline{8x}$$

$$2p - 12p$$

$$= \underline{-10p}$$

$$3x - 2y - 4x - 2y \quad \text{Listing}$$

$$\cancel{3x} - y - \cancel{4x} - y$$

$$\cancel{3x} - y - \cancel{4x} - y$$

$$\cancel{3x} - \cancel{4x} - x$$

$$= \underline{-x - 4y}$$

$$3m^2 - 3m + 3$$

$$= \underline{3m^2 - 3m + 3}$$

(no further simplification is possible)

$$3 \times y$$

$$= \underline{3y}$$

$$a \times a \times a$$

$$= \underline{a^3}$$

$$2b \times 4b \quad \text{expand}$$

$$= 2 \times 4 \times b \times b$$

$$= \underline{8b^2}$$


$$(-2b)^2$$

$$= -2b \times -2b$$

$$= -2 \times -2 \times b \times b$$


$$= \underline{4b^2}$$

Multiplying out Brackets - Claw!



$$3(a + 4)$$

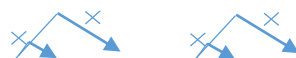
$$= 3a + 12$$



$$4 - 3(b - 5)$$

$$= 4 - 3b + 15$$


$$= \underline{19 - 3b}$$



$$3(a + b) + a(3 + b)$$

$$= 3a + 3b + 3a + ab$$

$$= \underline{6a + 3b + ab}$$



$$2a^2(b - 2) - a(a - 2ab)$$

$$= 2a^2b - 4a^2 - a^2 + 2a^2b$$

$$= \underline{4a^2b - 5a^2}$$

Equations

Examples:

$$b + 3 = 9$$

-3 -3

$$\underline{b = 6}$$

$$4 - d = 8$$

-4 -4

$$-d = 4$$

$$\underline{d = -4}$$

$$3x = 24$$

$$\frac{3x}{3} = \frac{24}{3}$$

$$\underline{x = 8}$$

$$\frac{e}{6} = 12$$

$$e = 12 \times 6$$

$$\underline{e = 72}$$



$$3m + 9 = 24$$

-9 -9

$$\frac{3m}{3} = \frac{15}{3}$$

$$\underline{m = 5}$$

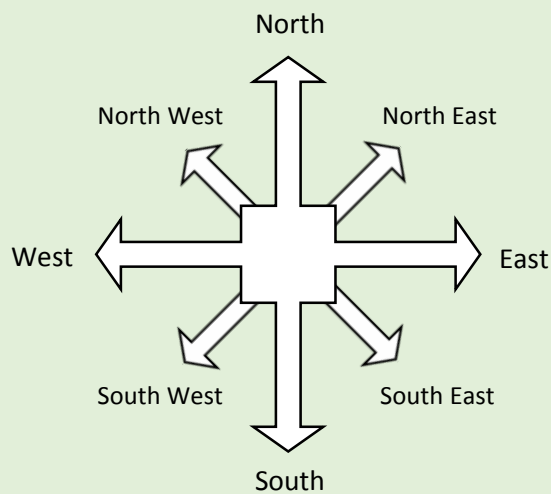
$$4n - 1 = 2$$

+1 +1

$$\frac{4n}{4} = \frac{3}{4}$$

$$n = \frac{3}{4}$$

Direction and the Compass

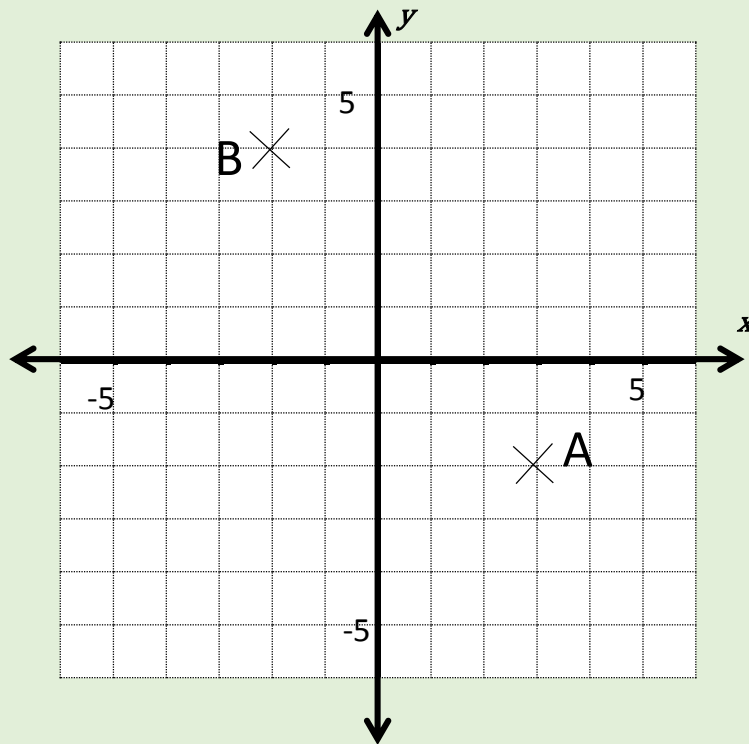


To remember use this mnemonic going clockwise from North:

Never Eat Shredded Wheat or **Naughty Elephants Spray Water**

Co-ordinate Grids

- A coordinate is used to specify a particular position on a grid:
- Point A is 3 along and 2 down or $A(3,-2)$
- Point B is 2 left and 4 up: $B(-2,4)$

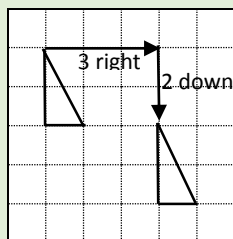


Transformations

- There are 4 types of transformations:

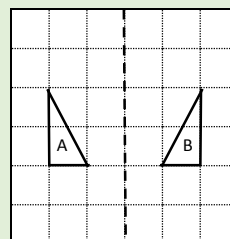
Translation

move the shape:



Reflection

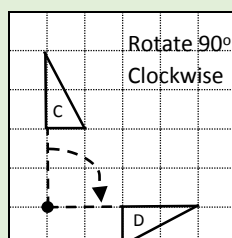
Make a mirror image:



B is a reflection of A

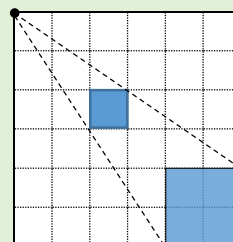
Rotation

Turn the shape:



Enlargement

Make the shape larger:

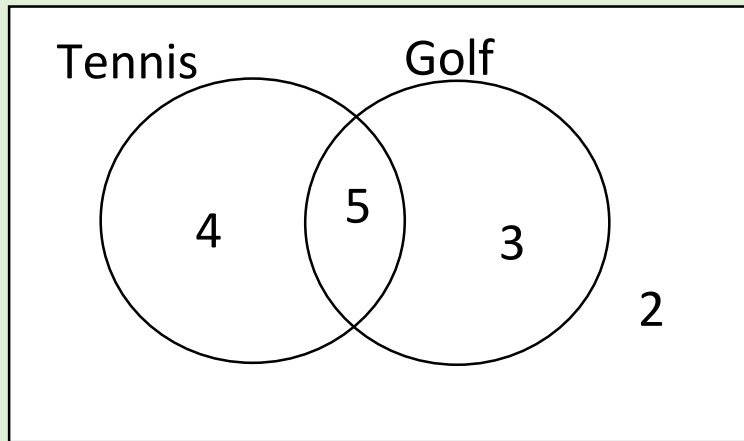


Enlarge shape E using a scale factor of 2 to make shape F

Venn Diagrams

- A Venn diagram is used to show information about sets of data.

The Venn diagram below shows the sports played by 14 children:



A total of 8 people play golf

A total of 9 people play tennis

7 people play only one sport

4 people play tennis only

5 people play both sports

3 people play golf only

2 people don't play either sport

Rounding

Decimal Places (d.p.)

- Decimal places are the digits which come after a decimal point.

Example:

3.155 has 3 decimal places: 3.155

- Example of rounding off to decimal places:

- Round 7.467 to 2 decimal places:

7.46:5

≈ 7.47 (the 6 changes since the digit to its right is 5 or greater)

Significant Figures (s.f.)

- Start counting significant figures from the first non-zero.

3.401 has 4 s.f. 0.044 has 2 s.f. 3183 has 4 s.f.

- Examples of rounding:

- 351 to 1 s.f. = 400

- 0.8483 to 3 s.f. = 0.848