



CARGILFIELD

Maths Revision

Book 2

Name:

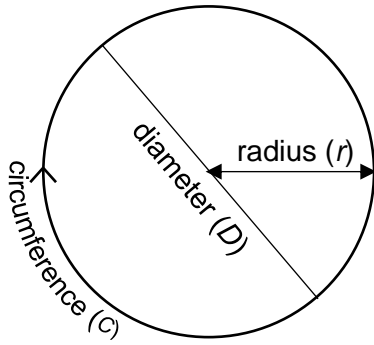
Fractions

Calculating with Fractions

<p style="text-align: center;">Addition</p> $\begin{array}{r} \frac{5}{6} + \frac{3}{4} \\ \xrightarrow{\times 2} \frac{10}{12} + \frac{9}{12} \\ \xrightarrow{\times 3} \frac{19}{12} \\ = 1\frac{7}{12} \end{array}$	<p style="text-align: center;">Subtraction</p> $\begin{array}{r} 3\frac{1}{3} - 1\frac{2}{3} \\ \text{Subtract wholes first} \\ = 2\frac{1}{3} - \frac{2}{3} \\ \text{Change one whole into thirds} \\ = 1 + \frac{3}{3} + \frac{1}{3} - \frac{2}{3} \\ = 1\frac{2}{3} \end{array}$
<p style="text-align: center;">Multiplication</p> $\begin{array}{r} 3\frac{3}{4} \times 1\frac{3}{5} \times \frac{3}{8} \\ = \frac{15}{4} \times \frac{8}{5} \times \frac{3}{8} \\ = \frac{\cancel{3}^3 \cancel{15}^1 \times \cancel{8}^1 \times 3}{4 \times \cancel{5}^1 \times \cancel{8}^1} \\ = \frac{9}{4} \\ = 2\frac{1}{4} \end{array}$	<p style="text-align: center;">Division</p> $\begin{array}{r} \frac{5}{12} \div \frac{15}{16} \\ \text{Invert right hand side} \\ = \frac{5}{12} \times \frac{16}{15} \\ = \frac{\cancel{5}^1 \times \cancel{16}^4}{\cancel{12}^3 \times \cancel{15}^3} \\ = \frac{4}{9} \end{array}$
Fraction Problems	
<p>Merry the monkey eats $1\frac{1}{3}$ bananas each day. How many bananas will Merry eat in 12 days?</p> $1\frac{1}{3} = \frac{4}{3} \quad \swarrow \times \frac{4}{3} \text{ bananas in 1 day } \times 3$ $\times 4 \left(\begin{array}{l} 4 \text{ bananas in 3 days} \\ \hline 16 \text{ bananas in 12 days} \end{array} \right) \times 4$	<p>How many days will 24 bananas last if Smiley eats $\frac{2}{5}$ of a banana each day?</p> $\swarrow \times \frac{2}{5} \text{ bananas in 1 day } \times 5$ $\times 12 \left(\begin{array}{l} 2 \text{ bananas in 5 days} \\ \hline 24 \text{ bananas in } \underline{60} \text{ days} \end{array} \right) \times 12$

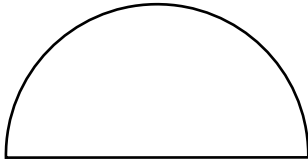
Shapes

Circles



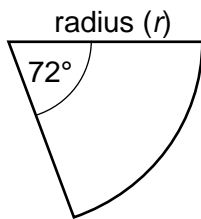
- Area of Circle = πr^2
- Circumference = $2\pi r$

Semi-circles



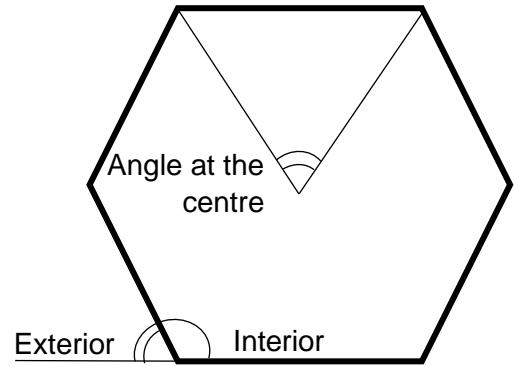
- Area = $\frac{1}{2} \pi r^2$
- Perimeter = $\frac{2\pi r}{2} + D$

Segments



- Find out how many segments make up a full circle:
e.g. $360^\circ \div 72^\circ = 5$
- Area of Segment (above) = $\frac{1}{5} \times \pi r^2$
- Perimeter = $\frac{1}{5} \times 2\pi r + 2r$

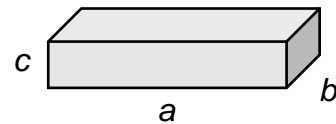
Polygons



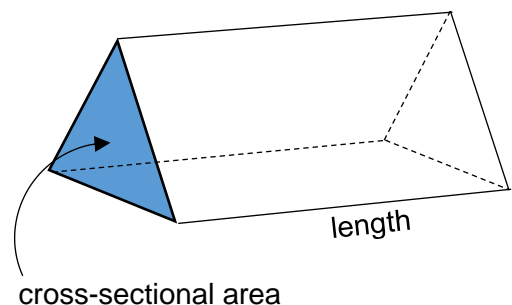
Polygon Formulae:

- Angle at the centre = Exterior angle
- Number of sides (n) = $360 \div$ Exterior angle
- Exterior angle = $360 \div$ Number of sides
- Angle at the centre = Exterior angle
- Interior angle + Exterior angle = 180°
- Sum of Interior angles = $180(n - 2)$

Volume



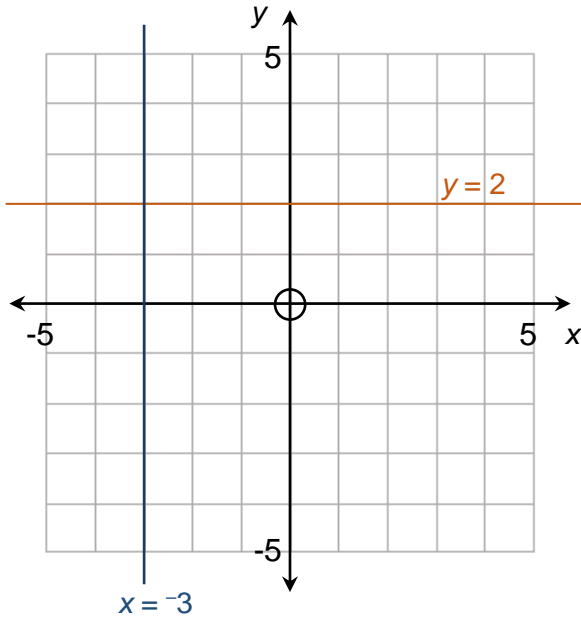
- Volume of a cuboid
= length x breadth x height
- Surface area of cuboid = $2ab + 2bc + 2ac$
- Volume of a prism
= cross-sectional area x length
- Surface area of a prism
= sum of the areas of all faces



Graphs

Straight Line Graphs

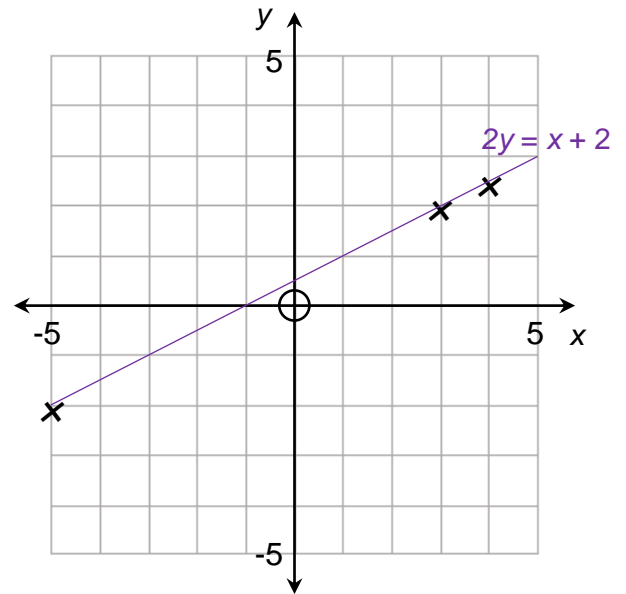
Horizontal and Vertical Graphs



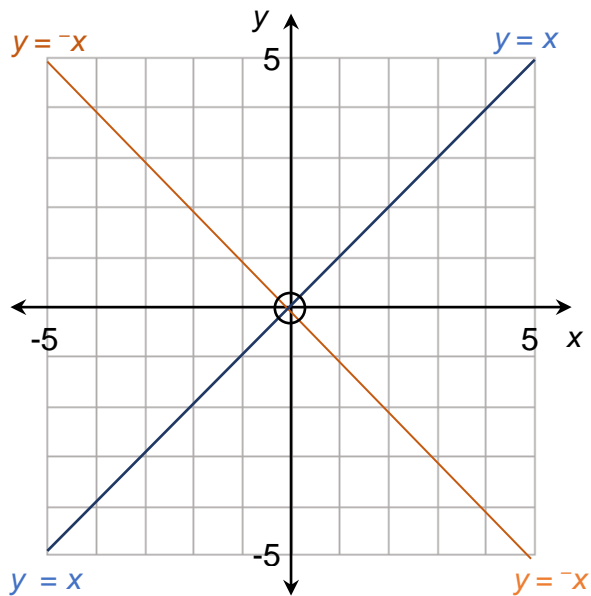
$y = mx + c$ type graphs

$2y = x + 1$ (which is same as $y = \frac{1}{2}x + \frac{1}{2}$)

x	-5	3	4
y	-2	2	2.5

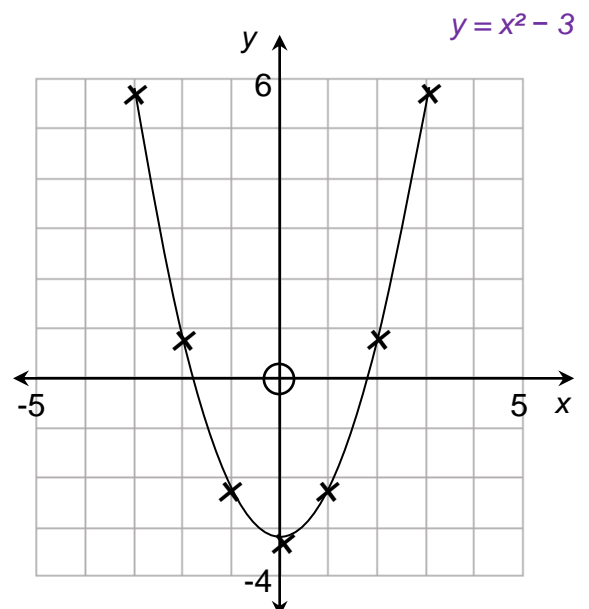


Diagonal Graphs



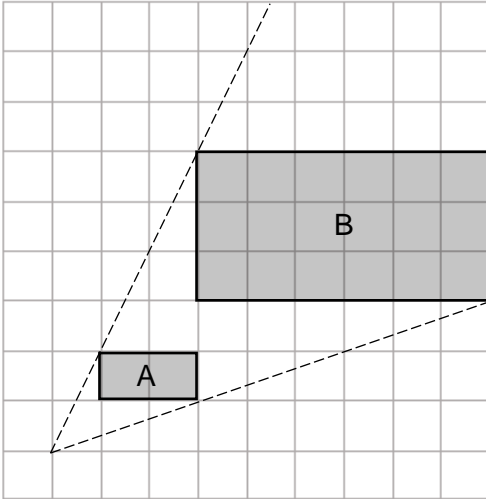
Quadratic Graphs (Level 3)

x	-3	-2	-1	0	1	2	3
y	6	1	-2	-3	-2	1	6



Enlargement: Area and Perimeter

- If the scale factor is 3, the perimeter of the enlarged shape will be 3 times longer.
- the area of the enlarged shape will be 9 times larger (3^2)



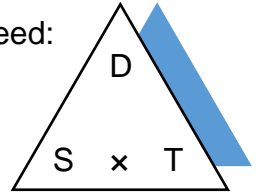
- Q. Triangle F is an enlargement of triangle E using a scale factor of 3.
If triangle F has an area of 72 cm^2 , find the area of triangle E.

$$\begin{aligned} \text{Area of E} &= 72 \div 3^2 \\ &= 72 \div 9 \\ &= \underline{8 \text{ cm}^2} \end{aligned}$$

Speed, Distance and Time

The formula to calculate speed:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$



- Work out the speed of a car travelling 180 miles in 3 hours and 20 minutes:

$$\text{Time in hours} = 3.33333333\dots \text{ or } 3\frac{20}{60}$$

$$\text{Speed} = \text{distance} \div \text{time}$$

$$= 180 \div 3.333333333 \left(3\frac{20}{60}\right)$$

$$= \underline{54 \text{ mph}}$$

- Work out the time taken to travel 96 km at a speed of 40 km/h:

$$\text{Time} = \text{distance} \div \text{speed}$$

$$= 96 \div 40$$

$$= 2.4 \text{ hours} = 2\frac{4}{10} = 2\frac{24}{60}$$

$$= \underline{2 \text{ hours } 24 \text{ minutes}}$$

- Change 4 m/s into km/h

$$\begin{array}{ccc} & \left(\begin{array}{c} \text{4 metres in 1 second} \end{array} \right) & \\ \times 3600 & \searrow & \swarrow \times 3600 \end{array}$$

$$14400 \text{ m in 1 hour}$$

$$= \underline{14.4 \text{ km/h}}$$

- Change 72 km/h into metres per second:

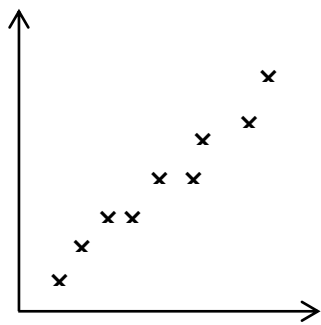
$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{72000 \text{ m}}{3600 \text{ s}}$$

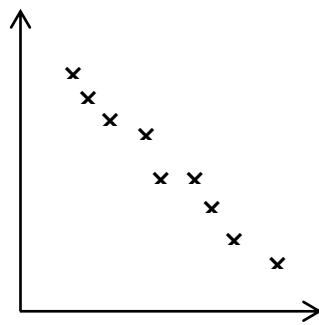
$$= \underline{20 \text{ m/s}}$$

*Remember: 1 hour = $60 \times 60 = 3600$ seconds

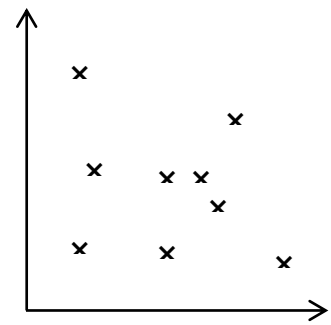
Scatter Graphs and Correlation



Positive Correlation



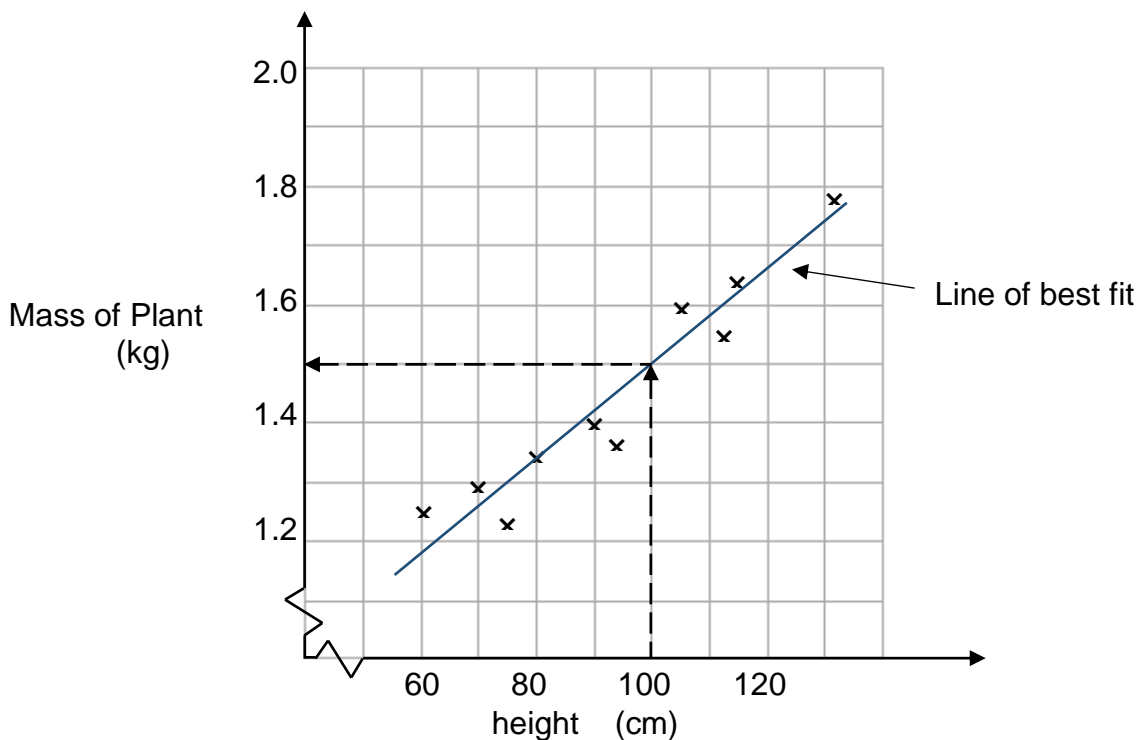
Negative Correlation



No Correlation

Example: Molly grows some plants and measures their height and mass. She then plots her results on a scatter graph as shown below.

Height and Mass of Plants



Question: Use the line of best fit to predict the mass of a 100 cm plant.

Answer: Show your working on the graph using dotted lines (see above).
If the plant has a height of 100 cm, it will weigh approximately 1.5 kg

Conversion Graphs

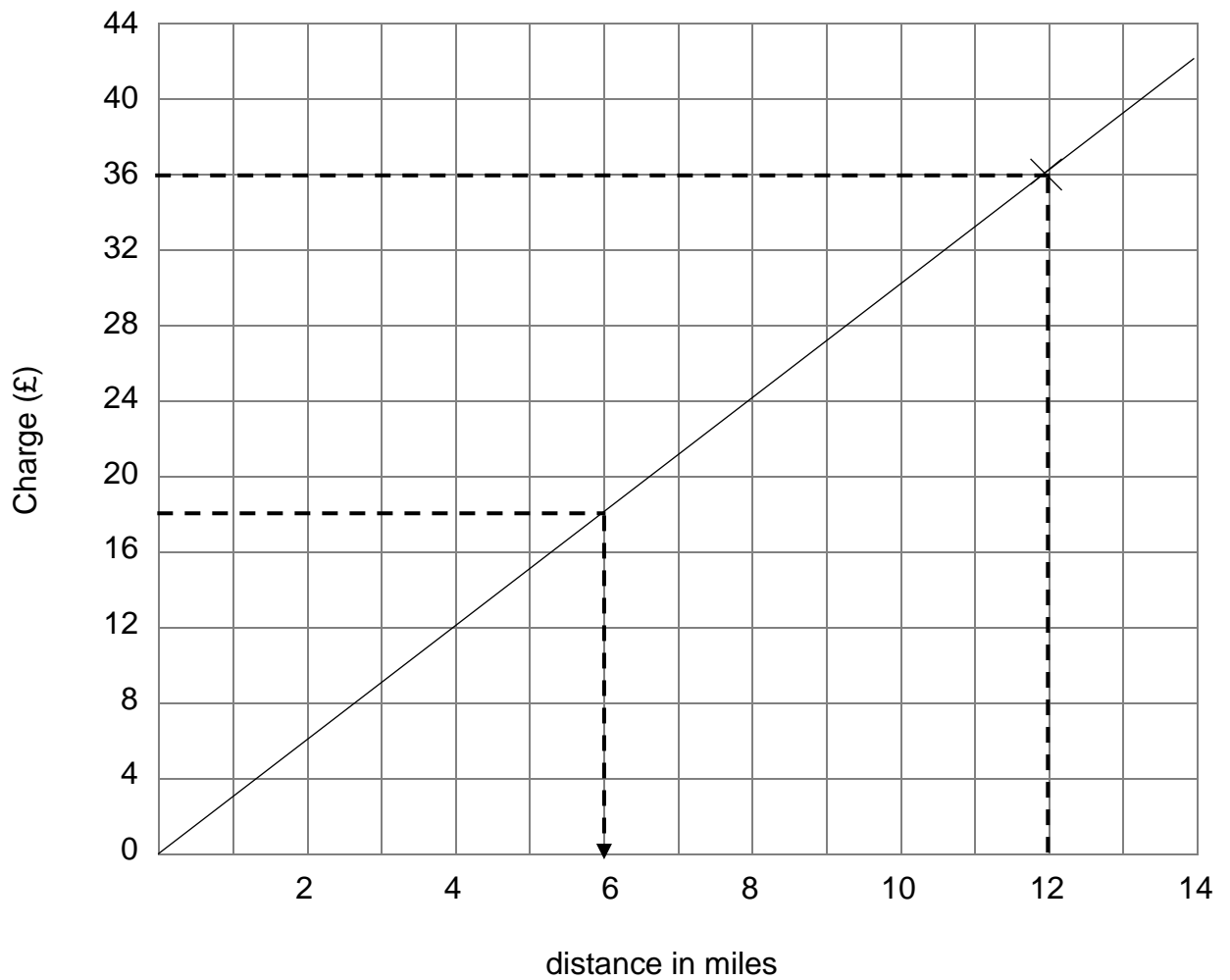
Example:

Easycab taxi company charge passengers £3 per mile.

How much would they charge a passenger for a 12 mile journey?

Answer:

$$\begin{array}{c} \times 12 \quad \left(\begin{array}{l} 1 \text{ mile is } \pounds 3 \\ 12 \text{ miles is } \pounds 36 \end{array} \right) \quad \times 12 \end{array}$$



If a passenger pays £18 for a journey, how far was the journey? Show your working on the graph above.

Answer: 6 miles.

Algebra

Simplification Examples:

(a)

$$\frac{3t + 18}{6}$$

$$= \frac{\overset{1}{\cancel{3}}t}{\underset{2}{\cancel{6}}} + \frac{18}{6}$$

$$= \frac{t}{2} + 3$$

(b)

$$(4y^3)^2$$

$$= 4y^3 \times 4y^3$$

$$= 4 \times 4 \times y \times y \times y \times y \times y \times y$$

$$= \underline{16y^6}$$

(c) $-15ab^2 \div 35a^2b^3$

$$= \frac{-15ab^2}{35a^2b^3}$$

$$= \frac{\overset{3}{\cancel{-15}} \times \cancel{a} \times \cancel{b} \times \cancel{b}}{\underset{7}{\cancel{35}} \times \cancel{a} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times \cancel{b}}$$

$$= -\frac{3}{7ab}$$

Using Factorisation:

(a) Fully factorise $8n + 36$

$$= \underline{4(2n + 9)}$$

(b) A square has a perimeter of $8n + 36$
Find the length of one side in terms of n

Use factorised expression: $4(2n + 9)$

Perimeter of a square = $4 \times$ length of side
= $4 \times (2n + 9)$

Length of one side = $2n + 9$

Substitution Examples:

(i) If $p = 3$, $r = 4$, $s = -2$, $t = 6$

Substitute:

(a) $2r - s$

$$= 2 \times 4 - (-2)$$

$$= 8 + 2$$

$$= \underline{10}$$

(b) $5p - r^2$

$$= 5 \times 3 - 4^2$$

$$= 15 - 16$$

$$= \underline{-1}$$

(c) $r(6s - p)$

$$= 4(6 \times -2 - 3)$$

$$= 4(-12 - 3)$$

$$= 4 \times (-15)$$

$$= \underline{-60}$$

(d) $\frac{pr - 8}{3t + s}$

$$= \frac{3 \times 4 - 8}{18 + -2}$$

$$= \frac{12 - 8}{18 - 2}$$

$$= \frac{4}{16}$$

$$= \frac{1}{4}$$

- (ii) If $v = \sqrt{5gh}$, work out the value of g
when $v = 20$ and $h = -10$

$$v = \sqrt{5gh}$$

$$20 = \sqrt{5 \times g \times -10}$$

$$400 = -50g \quad (\text{square both sides})$$

$$\underline{g = -8}$$

Equations Examples:

(a) $x + 3 = 1$
-3 -3

$$\underline{x = -2}$$

(b) $y - 4 = 6$
+4 +4

$$\underline{y = 10}$$

(c) $3y = 12$

$$\frac{3y}{3} = \frac{12}{3}$$

$$\underline{y = 4}$$

(d) $\frac{c}{5} = 3$ (or $\frac{1}{5}c = 3$)

$$\longleftarrow c \longrightarrow$$

3	3	3	3	3
---	---	---	---	---

$$\underline{c = 5 \times 3 = 15}$$

(e) $2a - 5 = 11$
+5 +5

$$2a = 16$$

$$\frac{2a}{2} = \frac{16}{2}$$

$$\underline{a = 8}$$

(f) $4 - 2x = 5 + x$

(take x's to side with biggest x)

$$4 - 2x = 5 + x$$

+2x +2x

$$4 = 5 + 3x$$

-5 -5

$$-1 = 3x$$

$$-\frac{1}{3} = \frac{3x}{3} \quad \begin{array}{l} / \\ / \end{array}$$

$$x = -\frac{1}{3}$$

(g) $\frac{2x}{3} = 8$

$$\longleftarrow 2x \longrightarrow$$

8	8	8
---	---	---

$$2x = 3 \times 8$$

$$2x = 24$$

$$\frac{2x}{2} = \frac{24}{2}$$

$$\underline{x = 12}$$

Brackets

$$\begin{array}{l} \text{(a)} \quad \begin{array}{c} \times \quad \times \\ \diagdown \quad \diagup \\ 2(8x + 6y) \end{array} \\ = \quad \underline{16x + 12y} \end{array}$$

$$\begin{array}{l} \text{(b)} \quad 5 - 3(4a + 2) \\ = \quad 5 - 12a - 6 \\ = \quad \underline{-1 - 12a} \end{array}$$

$$\begin{array}{l} \text{(c)} \quad 3(2m - 4n) - 2(m - 5n) \\ = \quad 6m - 12n - 2m + 10n \\ = \quad \underline{4m - 2n} \end{array}$$

Factorising

$$\begin{array}{l} \text{(a)} \quad 15f + 20 \\ = \quad \underline{5(3f + 4)} \end{array}$$

$$\begin{array}{l} \text{(b)} \quad 16a^2b - 24ab^3 \\ = \quad \underline{8ab(2a - 3b^2)} \end{array}$$

Algebra: Writing Expressions

On a train, there are m men, twice as many boys as men, 11 less girls than men, and three times as many women as girls.

In terms of m :

<u>Men</u>	<u>Boys</u>	<u>Girls</u>	<u>Women</u>
m	$2m$	$(m - 11)$	$3(m - 11)$

(a) Write an expression for the total number of people.

$$\begin{array}{l} m + 2m + (m - 11) + 3(m - 11) \\ = m + 2m + m - 11 + 3m - 33 \\ = \underline{7m - 44} \end{array}$$

(b) There are a total of 68 people on the train.
Write down and solve an equation to find the value of m .

$$\begin{array}{l} 7m - 44 = 68 \\ \quad +44 \quad +44 \\ 7m = 112 \\ \frac{7m}{7} = \frac{112}{7} \\ \underline{m = 16} \end{array}$$

(c) How many women are on the train?

$$\begin{array}{l} 3(m - 11) \\ = 3(16 - 11) \\ = 3 \times 5 \\ = \underline{15 \text{ women}} \end{array}$$

Number Patterns

Examples

Sequence	Rule	nth Term	20 th Term
7, 11, 15, 19, 23, ...	Add 4 each time (4 times table) ***See example at the bottom of the page***	$4n + 3$	$4n + 3$ $= 4 \times 20 + 3$ $= 83$
40, 35, 30, 25, 20, ..	Subtract 5 each time (based on -5 x table)	$-5n + 45$	$-5n + 45$ $= -5 \times 20 + 45$ $= -100 + 45$ $= -55$
1, 4, 9, 16, 25, ...	Square numbers	n^2	n^2 $= 20^2$ $= 400$
3, 6, 11, 18, 27, ...	Square then add 2	$n^2 + 2$	$n^2 + 2$ $= 20^2 + 2$ $= 402$
9, 16, 25, 36,	Add 2 then square	$(n + 2)^2$	$(n + 2)^2$ $= (20 + 2)^2$ $= 22^2$ $= 484$
2, 4, 8, 16, 32, ...	Doubling	2^n	8 th term: 2^n $= 2^8$ $= 256$
10, 100, 1000, ...	Multiply previous term by 10	10^n	8 th term: 10^n $= 10^8$ $= 100\ 000\ 000$

$$\begin{array}{ccccccc}
 & +4 & & +4 & & +4 & & +4 \\
 & \swarrow & & \swarrow & & \swarrow & & \swarrow \\
 7, & 11, & 15, & 19, & 23, & \dots & \rightarrow & 4n + 3 \\
 (& 4, & 8, & 12, & 16, & 20, & \dots) & \rightarrow & 4n
 \end{array}$$

The top sequence is the 4 times table but 3 more for each term,
i.e. $4n + 3$

Percentages

Find 35% of 360 m

Method 1:

35% of 360 m

$$\frac{35}{100} \times \frac{360}{1}$$

$$= \frac{\overset{7}{\cancel{35}} \times \overset{18}{\cancel{360}}}{\underset{5}{\cancel{100}} \underset{1}{\cancel{1}}}$$

$$= \frac{7 \times 18}{1}$$

$$= \underline{126 \text{ m}}$$

Method 2:

$$10\% = \frac{1}{10}$$

$$\frac{1}{10} \text{ of } 360 = 36 \text{ m}$$

$$10\% \rightarrow 36 \text{ m}$$

$$5\% \rightarrow 18 \text{ m}$$

$$30\% = 3 \times 36 \\ = 108 \text{ m}$$

$$35\% = 30\% + 5\% \\ = 108 + 18 \\ = \underline{126 \text{ m}}$$

Method 3 (Calculator)

$$35 \div 100 \times 360 \\ = \underline{126 \text{ m}}$$

Method 4 (Calculator)

Buttons on calculator:

$$\boxed{3}\boxed{5}\boxed{\%}\boxed{\times}\boxed{3}\boxed{6}\boxed{0}$$

Pie Charts

Angle for 1 Person

30 people surveyed.

$$\text{Angle for 1 person} = 360^\circ \div 30 \\ = \underline{12^\circ}$$

Percentage pie charts:

$$1\% = \frac{360}{100} = \underline{3.6^\circ}$$

Bearings

You must use 3 figures.

$$042^\circ, \quad 009^\circ, \quad 316^\circ \\ \uparrow \quad \quad \uparrow\uparrow$$

Write the bearings and real life distances on the diagrams.

Scales

$$1 : 10\,000$$

$$1 : 500$$

to change these do: -

$$1 \text{ cm} : 10\,000 \text{ cm}$$

$$1 \text{ cm} : 500 \text{ cm}$$

$$\underline{1 \text{ cm} : 100 \text{ m}}$$

$$\underline{1 \text{ cm} : 5 \text{ m}}$$