CARGILFIELD

DEO CUSTODE

Maths Revision

Book 2

Name: ……………………………………..

Fractions

Calculating with Fractions

Addition

$$\frac{5}{6} + \frac{3}{4}$$

×2   ×2   ×3   ×3

$$= \frac{10}{12} + \frac{9}{12}$$

$$= \frac{19}{12}$$

$$= 1\frac{7}{12}$$

Subtraction

$$3\frac{1}{3} - 1\frac{2}{3}$$

Subtract wholes first

$$= 2\frac{1}{3} - \frac{2}{3}$$

Change one whole into thirds

$$= 1 + \frac{3}{3} + \frac{1}{3} - \frac{2}{3}$$

$$= 1\frac{2}{3}$$

Multiplication

$$3\frac{3}{4} \times 1\frac{3}{5} \times \frac{3}{8}$$

$$= \frac{15}{4} \times \frac{8}{5} \times \frac{3}{8}$$

$$= \frac{^{3}\cancel{15} \times {}^{1}\cancel{8} \times 3}{4 \times \cancel{5} \times \cancel{8}}$$
$$\quad\quad\quad {}_{1} \quad\quad {}_{1}$$

$$= \frac{9}{4}$$

$$= 2\frac{1}{4}$$

Division

$$\frac{5}{12} \div \frac{15}{16}$$

Invert right hand side

$$= \frac{5}{12} \times \frac{16}{15}$$

$$= \frac{{}^{1}\cancel{5} \times {}^{4}\cancel{16}}{\cancel{12} \times \cancel{15}}$$
$$\quad\quad {}_{3} \quad\quad {}_{3}$$

$$= \frac{4}{9}$$

Fraction Problems

Merry the monkey eats 1⅓ bananas each day. How many bananas will Merry eat in 12 days?

$$1\frac{1}{3} = \frac{4}{3}$$ $\cancel{3} \times \frac{4}{\cancel{3}}$ bananas in 1 day × 3

×4 ⟮ 4 bananas in 3 days ⟯ ×4

16 bananas in 12 days
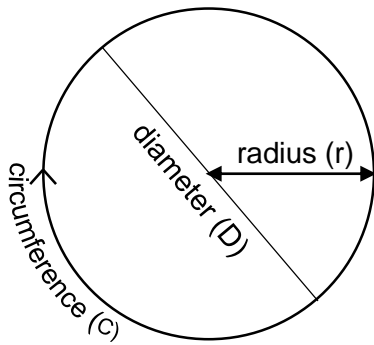
How many days will 24 bananas last if Smiley eats $\frac{2}{5}$ of a banana each day?

$\cancel{5} \times \frac{2}{\cancel{5}}$ bananas in 1 day × 5

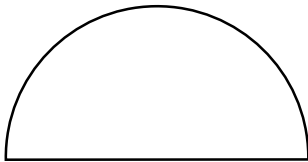×12 ⟮ 2 bananas in 5 days ⟯ ×12

24 bananas in 60 days

Shapes

Circles
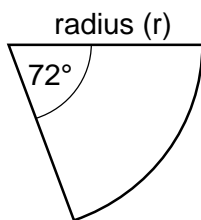

- Area of Circle = πr^2
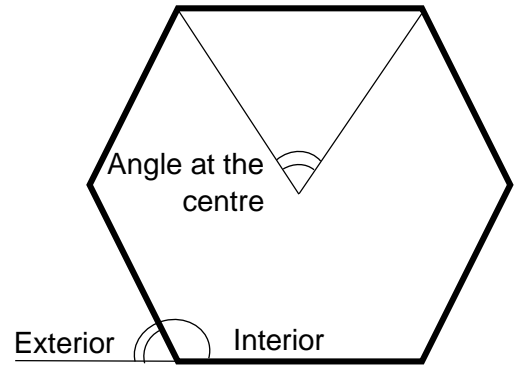- Circumference = $2\pi r$

Semi-circles

- Area = $\frac{1}{2}\pi r^2$

- Perimeter = $\frac{2\pi r}{2} + D$

Segments

- Find out how many segments make up a full circle:
  e.g. $360° \div 72° = 5$

- Area of Segment (above) = $\frac{1}{5} \times 2\pi r$

- Perimeter = $\frac{1}{5} \times 2\pi r + 2r$

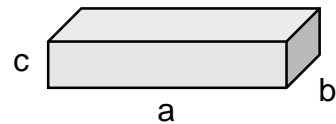## Polygons

Polygon Formulae:

- Number of sides(n) = 360 ÷ Exterior angle
- Exterior angle = 360 ÷ Number of sides
- Angle at the centre = Exterior angle
- Interior angle + Exterior angle = 180^0
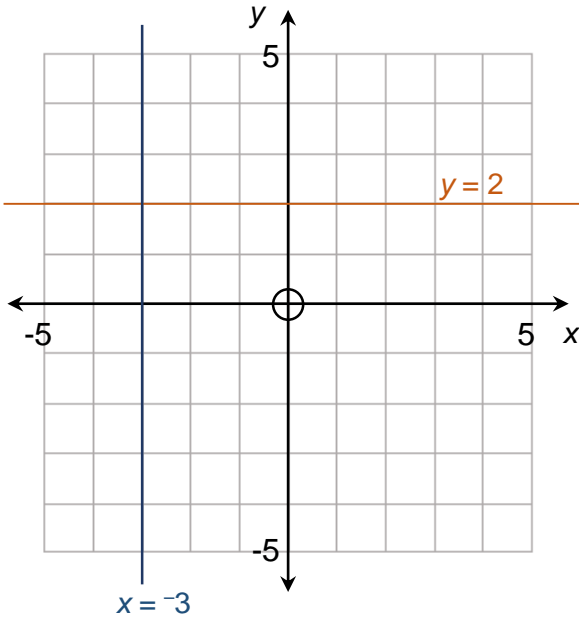- Sum of Interior angles = 180(n – 2)

Volume

- Volume of a cuboid
  = length x breadth x height

- Surface area of cuboid = 2ab + 2bc + 2ac

- Volume of a prism
  = cross-sectional area × length

- Surface area of a prism
  = sum of the areas of all faces

Cross-sectional area
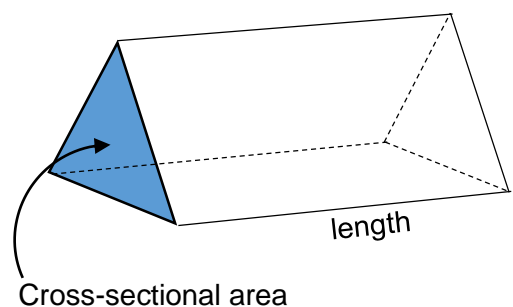
Straight Line Graphs

Horizontal and Vertical Graphs

$y = 2$

$x = {}^-3$

$y = mx + c$ type graphs

$2y = x + 1$ (which is same as $y = \frac{1}{2}x + \frac{1}{2}$)

x	-5	3	4
y	${}^-2$	2	2.5

$2y = x + 2$

Diagonal Graphs

$y = {}^-x$

$y = x$

$y = x$

$y = {}^-x$
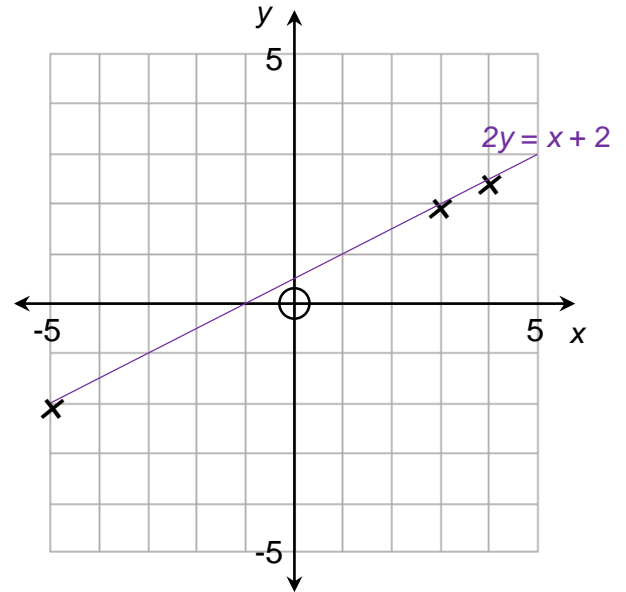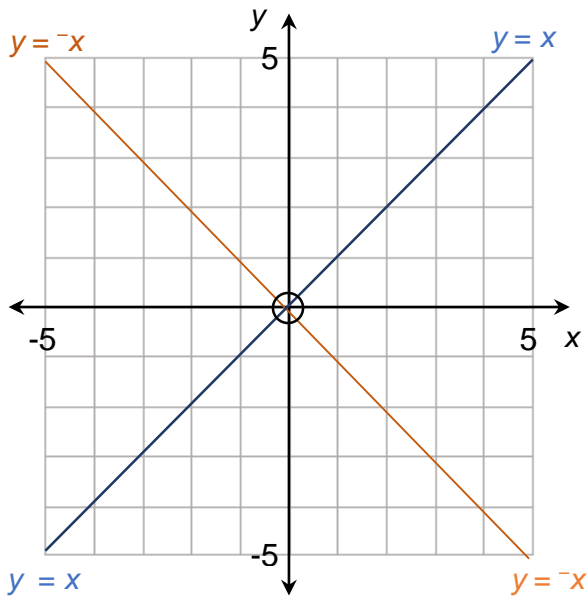
Quadratic Graphs (Level 3)

x	${}^-3$	${}^-2$	${}^-1$	0	1	2	3
y	6	1	${}^-2$	${}^-3$	${}^-2$	1	6

$y = x^2 - 3$
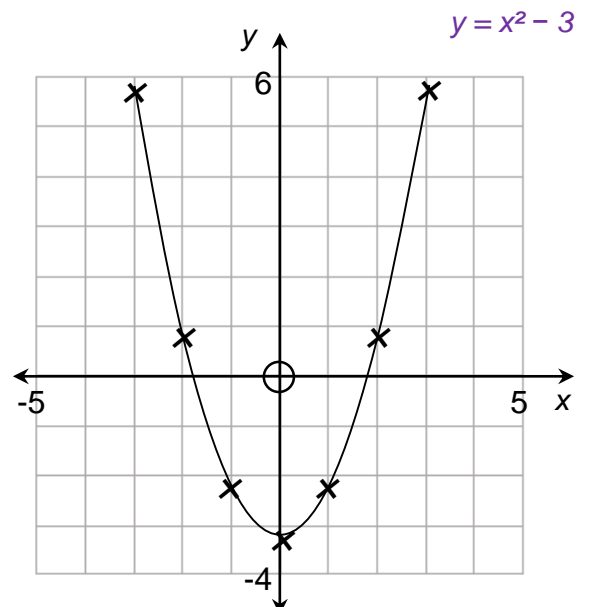
Enlargement: Area and Perimeter

- If the scale factor is 3, the perimeter of the enlarged shape will be 3 times longer.
- the area of the enlarged shape will be 9 times larger (3^2)
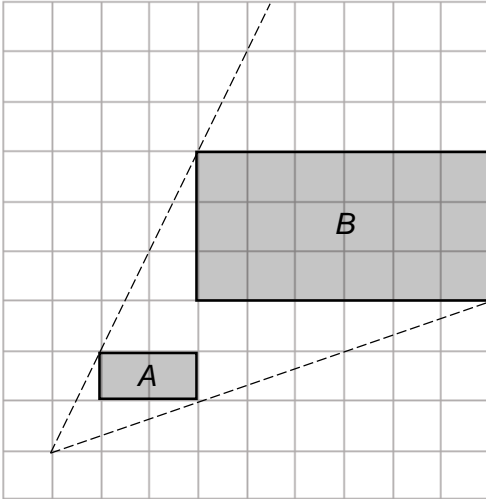
Q. Triangle F is an enlargement of triangle E using a scale factor of 3.
If triangle F has an area of 72 cm², find the area of triangle E.

Area of E = 72 ÷ 3^2
= 72 ÷ 9
= 8 cm²

Speed, Distance and Time

The formula to calculate speed:

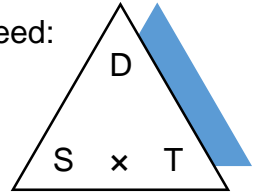$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

- Work out the speed of a car travelling 180 miles in 3 hours and 20 minutes:

Time in hours = 3.33333333… or $3\frac{20}{60}$

Speed = distance ÷ time

= 180 ÷ 3.333333333 $(3\frac{20}{60})$

= 54 mph

- Work out the time taken to travel 96 km at a speed of 40 km/h:

Time = distance ÷ speed
= 96 ÷ 40
= 2.4 hours = $2\frac{4}{10}$ = $2\frac{24}{60}$

= 2 hours 24 minutes

- Change 4 m/s into km/h

× 3600   4 metres in 1 second   × 3600

14400 m in 1 hour

= 14.4 km/h

- Change 72 km/h into metres per second:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{72000 \text{ m}}{3600 \text{ s}}$$

= 20 m/s

*Remember: 1 hour = 60 × 60 = 3600 seconds

Scatter Graphs and Correlation
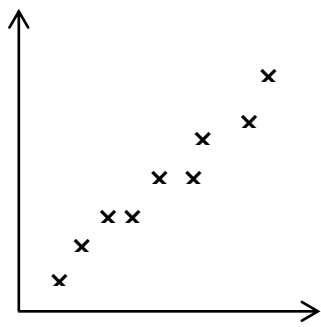
Positive Correlation   Negative Correlation   No Correlation
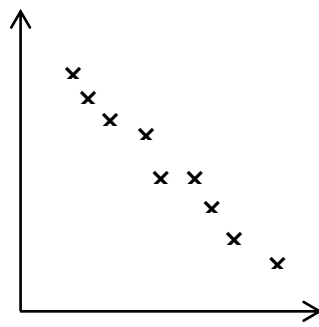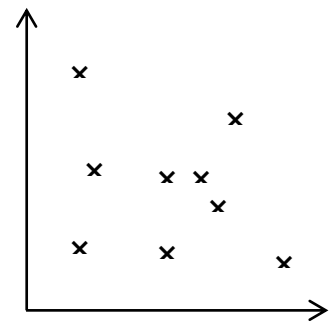
Example: Molly grows some plants and measures their height and mass.
             She then plots her results on a scatter graph as shown below.

Height and Mass of Plants
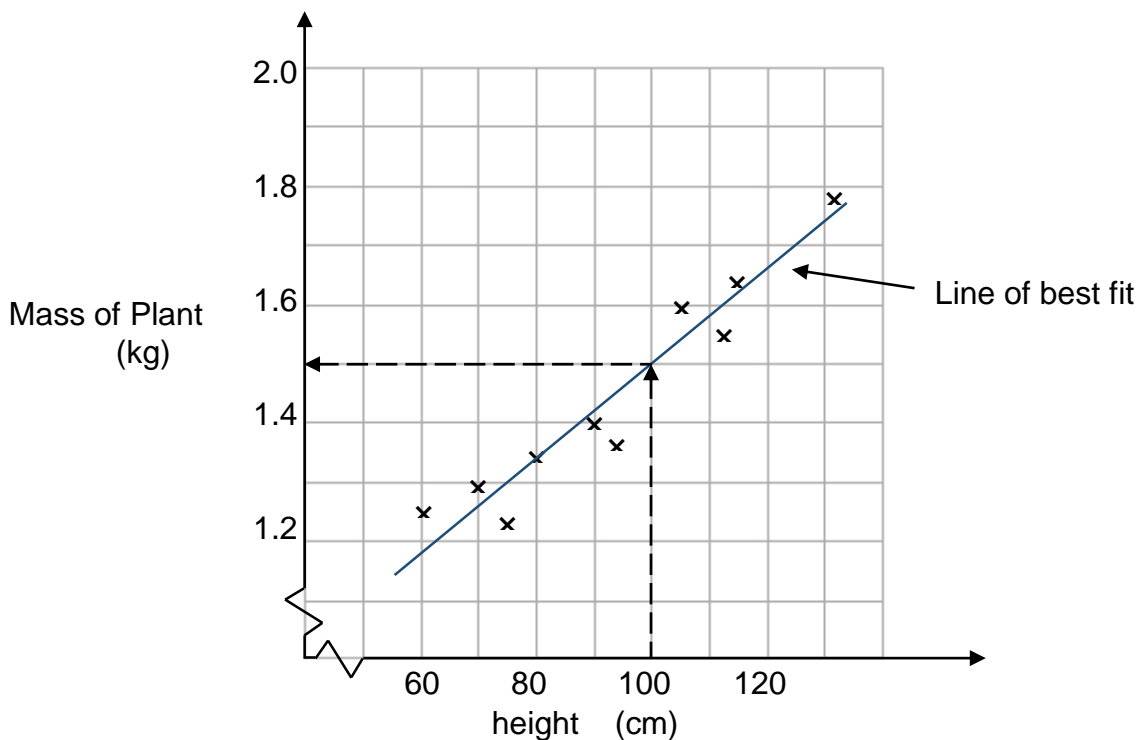
Question: Use the line of best fit to predict the mass of a 100 cm plant.

Answer: Show your working on the graph using dotted lines (see above).
        If the plant has a height of 100 cm, it will weigh approximately 1.5 kg

Conversion Graphs

Example:

Easycab taxi company charge passengers £3 per mile.
How much would they charge a passenger for a 12 mile journey?

Answer :

× 12 (1   mile is £3) × 12
          12 miles is £36

Charge (£)

distance in miles

If a passenger pays £18 for a journey, how far was the journey? Show your working on the graph above.

Answer: 6 miles.

Algebra

Simplification Examples:

(a)

$$\frac{3t + 18}{6}$$

$$= \frac{\overset{1}{\cancel{3}}t}{\underset{2}{\cancel{6}}} + \frac{18}{6}$$

$$= \frac{t}{2} + 3$$

(b)

$$(4y^3)^2$$

$$= 4y^3 \times 4y^3$$

$$= 4 \times 4 \times y \times y \times y \times y \times y \times y$$

$$= \underline{16\,y^6}$$

(c) $^-15ab^2 \div 35a^2b^3$

$$= \frac{^-15ab^2}{35a^2b^3}$$

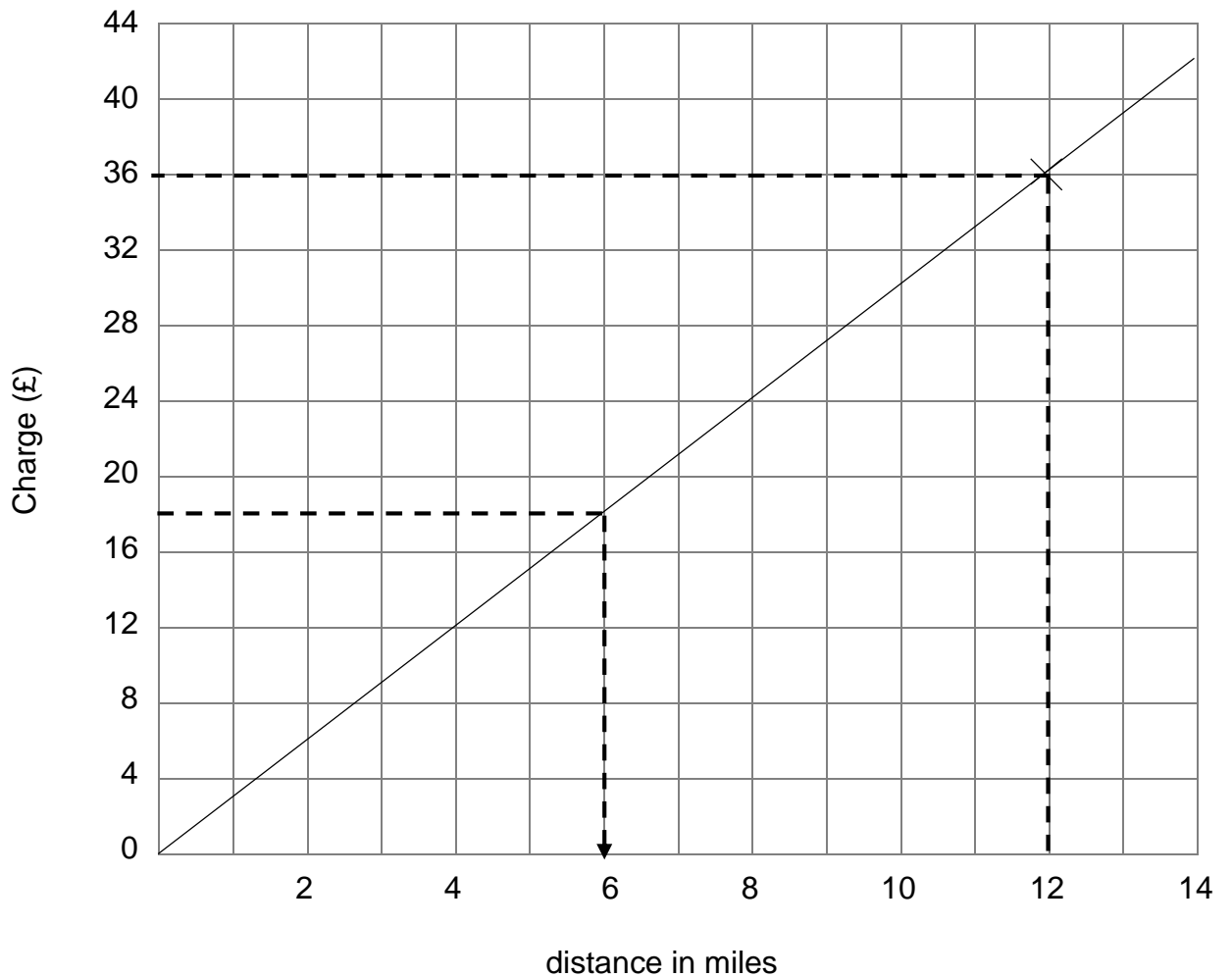$$= \frac{\overset{3}{^-\cancel{15}} \times a \times \cancel{b} \times \cancel{b}}{\underset{7}{\cancel{35}} \times \cancel{a} \times a \times \cancel{b} \times \cancel{b} \times b}$$

$$= -\frac{3}{7ab}$$

Using Factorisation:

(a)   Fully factorise $8n + 36$

$$= \underline{4(2n + 9)}$$

(b) A square has a perimeter of $8n + 36$
     Find the length of one side in terms of n

Use factorised expression: $4(2n + 9)$

Perimeter of a square = 4 × length of side
                      = 4 × (2n + 9)

Length of one side = $\underline{2n + 9}$

Substitution Examples:

(i) If $p = 3$, $r = 4$, $s = {}^-2$, $t = 6$

Substitute:

(a) $2r - s$
        $= 2 \times 4 - (^-2)$
        $= 8 + 2$
        $= \underline{10}$

(b) $5p - r^2$
        $= 5 \times 3 - 4^2$
        $= 15 - 16$
        $= \underline{^-1}$

(c) $r(6s - p)$
        $= 4(6 \times {}^-2 - 3)$
        $= 4(^-12 - 3)$
        $= 4 \times (^-15)$
        $= \underline{^-60}$

(d) $\frac{pr - 8}{3t + s}$

$$= \frac{3 \times 4 - 8}{18 + {}^-2}$$

$$= \frac{12 - 8}{18 - 2}$$

$$= \frac{4}{16}$$

$$= \frac{1}{4}$$

(ii) If $v = \sqrt{5gh}$, work out the value of g when $v = 20$ and $h = {}^{-}10$

$$v = \sqrt{5gh}$$

$$20 = \sqrt{5 \times g \times \text{-}10}$$

$$400 = {}^{-}50g \qquad \textit{(square both sides)}$$

$$\underline{g = {}^{-}8}$$

Equations Examples:

(a) $x + 3 = 1$
 -3 -3

$$\underline{x = {}^{-}2}$$

(b) $y - 4 = 6$
 +4 +4

$$\underline{y = 10}$$

(c) $3y = 12$

$$\frac{\cancel{3}y}{\cancel{3}} = \frac{12}{3}$$

$$\underline{y = 4}$$

(d) $\dfrac{c}{5} = 3$ (or $\tfrac{1}{5}c = 3$)

$$\underline{c = 5 \times 3 = 15}$$

(e) $2a - 5 = 11$
 +5 +5

$$2a = 16$$

$$\frac{\cancel{2}a}{\cancel{2}} = \frac{16}{2}$$

$$\underline{a = 8}$$

(f) $4 - 2x = 5 + x$

(take x's to side with biggest x)

$4 - 2x = 5 + x$
 +2x +2x

$4 = 5 + 3x$
-5 -5

$${}^{-}1 = 3x$$

$$\frac{{}^{-}1}{3} = \frac{\cancel{3}x}{\cancel{3}}$$
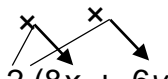
$$x = {}^{-}\frac{1}{3}$$

(g) $\dfrac{2x}{3} = 8$

$$2x = 3 \times 8$$

$$2x = 24$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{24}{2}$$

$$\underline{x = 12}$$

Brackets

(a)   2 (8x + 6y)

=   16x + 12y

(b)   5 − 3 (4a + 2)

=   5 − 12a − 6

=   ⁻1 − 12a

(c)   3(2m − 4n) − 2(m − 5n)

=   6m − 12n − 2m + 10n

=   4m − 2n

Factorising

(a)   15f + 20

=   5 (3f + 4)

(b)   16a²b − 24ab³

=   8ab(2a − 3b²)

Algebra: Writing Expressions

On a train, there are *m* men, twice as many boys as men, 11 less girls than men, and three times as many women as girls.

In terms of *m*:

Men	Boys	Girls	Women
m	2m	(m − 11)	3(m − 11)

(a) Write an expression for the total number of people.

   m + 2m + (m − 11) + 3(m − 11)

   = m + 2m + m − 11 + 3m − 33

   = 7m − 44

(b)   There are a total of 68 people on the train.
   Write down and solve an equation to find the value of *m*.

   7m − 44 = 68
      +44   +44

       7m = 112

       $\frac{7m}{7}$ = $\frac{112}{7}$

       m = 16

(c)   How many women are on the train?

       3(m − 11)
       = 3(16 − 11)
       = 3 × 5
       = 15 women

Number Patterns

Examples

Sequence	Rule	nth Term	20th Term
7, 11, 15, 19, 23, …	Add 4 each time (4 times table) ***See example at the bottom of the page***	$4n + 3$	$4n + 3$ $= 4 \times 20 + 3$ $= 83$
40, 35, 30, 25, 20, ..	Subtract 5 each time (based on -5 × table)	$-5n + 45$	$-5n + 45$ $= {}^-5 \times 20 + 45$ $= {}^-100 + 45$ $= {}^-55$
1, 4, 9, 16, 25, …	Square numbers	n^2	n^2 $= 20^2$ $= 400$
3, 6, 11, 18, 27, …	Square then add 2	$n^2 + 2$	$n^2 + 2$ $= 20^2 + 2$ $= 402$
9, 16, 25, 36, ….	Add 2 then square	$(n + 2)^2$	$(n + 2)^2$ $= (20 + 2)^2$ $= 22^2$ $= 484$
2, 4, 8, 16, 32, …	Doubling	2^n	8th term: 2^n $= 2^8$ $= 256$
10, 100, 1000, …	Multiply previous term by 10	10^n	8th term: 10^n $= 10^8$ $= 100\ 000\ 000$

$$\begin{array}{ccccc} \overset{+4}{\frown} & \overset{+4}{\frown} & \overset{+4}{\frown} & \overset{+4}{\frown} \\ 7, & 11, & 15, & 19, & 23, \dots \rightarrow \quad \mathbf{4n + 3} \\ (\ 4, & 8, & 12, & 16, & 20, \dots) \rightarrow \quad \mathbf{4n} \end{array}$$

The top sequence is the 4 times table but 3 more for each term,

i.e. $4n + 3$

<u>Percentages</u>

Find 35% of 360 m

Method 1:

35% of 360 m

$$\frac{35}{100} \times \frac{360}{1}$$

$$= \frac{\overset{7}{\cancel{35}} \times \overset{18}{\cancel{360}}}{\underset{\underset{1}{5}}{\cancel{100}}}$$

$$= \frac{7 \times 18}{1}$$

$$= \underline{126\,m}$$

Method 2:

$$10\% = \frac{1}{10}$$

$$\frac{1}{10} \text{ of } 360 = 36\,m$$

$$10\% \rightarrow 36\,m$$

$$5\% \rightarrow 18\,m$$

$$30\% = 3 \times 36$$
$$= 108\,m$$

$$35\% = 30\% + 5\%$$
$$= 108 + 18$$
$$= \underline{126\,m}$$

Method 3 (Calculator)

$$35 \div 100 \times 360$$
$$= \underline{126\,m}$$

Method 4 (Calculator)

Buttons on calculator:
$\boxed{3}\boxed{5}$ $\boxed{\%}$ $\boxed{\times}$ $\boxed{3}\boxed{6}\boxed{0}$

Pie Charts

Angle for 1 Person

30 people surveyed.
Angle for 1 person = 360° ÷ 30
$$= \underline{12°}$$

Percentage pie charts:

$$1\% = \frac{360}{100} = \underline{3.6°}$$

<u>Bearings</u>

You must use 3 figures.

042°,   009°,   316°
↑   ↑↑

Write the bearings and real life distances on the diagrams.

Scales

1 : 10 000   1 : 500

to change these do: -

1 cm : 10 000 cm   1 cm : 500 cm

<u>1 cm : 100 m</u>   <u>1 cm : 5 m</u>